# Computer Systems Group

## The Development and Evaluation of a Model of Visual Acuity for Computer-Generated Imagery

by

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# The Development and Evaluation of a Model of Visual Acuity for Computer-Generated Imagery

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**Abstract:** This paper presents a model for human visual acuity which accommodates the effects of peripheral vision and target velocity. The model is based upon work from the field of visual perception in order to provide an accurate and principled result. The model is evaluated through a number of psychophysical experiments and subsequently refined for the genus of stimuli commonly found in computer graphics applications. With this work, and the companion paper Reddy (1996), computer graphics developers gain a reliable method to predict the ability of an average user to perceive detail at any point in a computer-generated scene.

**Keywords:** contrast sensitivity, psychophysics, spatial frequency, threshold vision, visual acuity.

## 1 Introduction

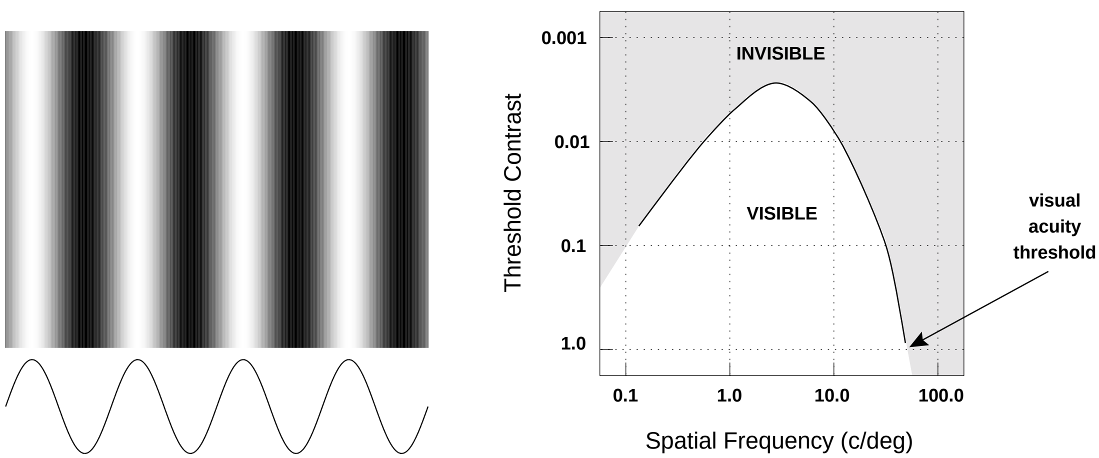
This paper is concerned with the task of predicting the visibility of any arbitrary stimulus on a computer display device. That is, to resolve whether a user can perceive a certain degree of detail based upon the extent to which it exists in their peripheral field and the angular velocity with which it is moving across their retinae. One possible application of this measure would be to remove any extraneous visual detail from a scene that a user cannot perceive; and therefore reduce the computational burden on the computer graphics system.

It is well known that our ability to perceive detail degrades with the retinal eccentricity (distance into the peripheral field) and the angular velocity of a target. Many computer graphics researchers have therefore attempted to take advantage of this phenomenon by reducing the spatial resolution in a scene where the user is less visually sensitive (e.g. Levoy and Whitaker, 1990; Funkhouser and Séquin, 1993; Hitchner and McGreevy, 1993; Ohshima *et al.*, 1996). However, most of these systems have been only superficially based upon any knowledge of human visual performance. As a result, these systems tend to introduce *ad hoc* models; incorporating various arbitrary variables which can only be instantiated through numerous trial and error procedures. We assert here that any system attempting to make judgements on a user's percept must be founded on principled models of visual perception. The model that we will present will therefore be developed with strict adherence to contemporary theories and practices in the field of visual perception.

This work can be considered continuous to that appearing in Reddy (1996), which attempted to provide a means to quantify the degree of visual detail in a computer-generated image. The aforementioned article provides us with a definition for the extent of a visual feature in an image: while this work will confer an ability to decide whether or not a user can perceive such a feature under various viewing conditions. Acquaintance with this previous paper would be advantageous.

## 2 Background

In Reddy (1996) we introduced the *contrast grating*: a simple harmonic pattern which is used by vision scientists to assess the ability of an individual to resolve detail (see Figure 1(a)). A contrast grating can be characterised by its two principal determinants, *contrast* and *spatial frequency*. Contrast is simply a measure of the luminance difference between adjacent light and dark bars; whereas spatial frequency is a measure of the spacing between bars, defined in units of contrast cycles per degree of visual field (c/deg). For example, a high spatial frequency implies a short distance between adjacent bars and hence represents a stimulus of high detail.




**(a)** **(b)**

Figure 1: *(a) an example contrast grating. The curve below the grating shows the sinusoidal nature of the intensity distribution. If this grating was positioned to occupy 1 deg of visual arc, then it would have a spatial frequency of 4 c/deg. (b) an example contrast sensitivity function for static detail.*

To discover a user's threshold vision for a grating of a particular spatial frequency, the observer alters the grating's contrast until they can no longer perceive the discrete bars of the pattern. For a number of such experiments we can plot a user's *threshold contrast* (0–1) as a function of spatial frequency. The resultant curve is referred to as a *contrast sensitivity function*, or CSF (see Figure 1(b)). The CSF describes an observer's window of visibility, i.e. the range of detail which they can perceive.

It is also common in the vision field to refer to an individual's *contrast sensitivity*. This is simply defined as the reciprocal of a person's threshold contrast. Another useful measure is *visual acuity*, which is defined as the smallest detail that a person can resolve (i.e. this is only a measure of size and does not take into consideration the contrast of a target).

In this paper we will only be concerned with predicting a user's visual acuity under various motive and eccentric conditions. However, little work has been done to quantify visual acuity in anything other than static, foveal[1] conditions. It is possible however to derive visual acuity from contrast sensitivity: given the definition of visual acuity above, we can see that this corresponds to the upper point where the CSF meets the x-axis; because below this point all stimuli are visible at full contrast, but beyond this point no stimuli are visible, irrespective of their contrast. Figure 1(b) illustrates this relationship. We will therefore build our model of visual acuity on a suitable model for contrast sensitivity.

# 3 Developing the Model

Kelly (1975) formulated a mathematical system to describe the shape of the human CSF. He developed a conceptual model to describe the spatial frequency characteristics of retinal receptive fields at high illuminance levels, and showed that this can be used to model the sensitivity of the visual system to sinusoidal stimuli such as contrast gratings. Kelly's abstract model for contrast sensitivity is defined as:

$$F(\alpha) = \alpha^2 \exp(-\alpha), \tag{1}$$

where $\alpha$ represents spatial frequency (c/deg). (Recall that contrast sensitivity is defined as the reciprocal of threshold contrast, i.e. threshold contrast can be modeled by $1/F(\alpha)$.)

With suitable scaling factors, this general equation can be used to model the shape of the CSF under various viewing conditions. For our purposes, we wish to take into consideration the effects of velocity and eccentricity. We will therefore investigate how these variables can be incorporated into Equation 1.

## 3.1 Incorporating Velocity into the Model

The surface which is produced by mapping the CSF for a range of velocities is called the *spatiotemporal threshold surface*. This has been investigated by a number of vision researchers over the years. Our model will be based upon the work of D. H. Kelly; although it is worth noting that Burr and Ross (1982) conducted similar experiments to those of Kelly, and that their principal results correlate almost exactly with his.

Kelly (1979) made extensive studies of the spatiotemporal surface. From his data, he noted that the shape of the CSF remains essentially constant for all velocities above 0.1 deg/s; and only undergoes translation with increased velocity. He subsequently extended Equation 1 to

[1]The fovea is the part of the retina with the highest sensitivity to spatial detail, i.e. 0 deg eccentricity.

model the spatiotemporal threshold surface (for velocities above 0.1 deg/s), by introducing two scale factors: $k$ (the height of the CSF), and $\alpha_{\max}$ (the peak frequency of the CSF). His equation can be presented as follows:

$$G(\alpha, v) = kv\alpha^2 \exp(-2\alpha/\alpha_{\max}), \tag{2}$$

where,

$$k = 6.1 + 7.3|\log_{10}(v/3)|^3, \tag{3}$$

$$\alpha_{\max} = 45.9/(v+2). \tag{4}$$

Where $v$ represents velocity measured in units of deg/s, and $\alpha$ represents spatial frequency in units of c/deg. Equations 2–4 can be combined into a single expression to give:

$$G(\alpha, v) \quad = \quad [6.1 + 7.3|\log_{10}(v/3)|^3]v\alpha^2 \exp[-2\alpha(v+2)/45.9]. \tag{5}$$

However, the author was unable to reproduce the empirical data which Kelly (and others) presents using this formula. Therefore, a modified version of Equation 5 was developed to more accurately model the available data. This was determined empirically with computer-aided techniques. The final solution was obtained by effectively altering the weighting of the $k$ and $\alpha_{\max}$ components (with weightings of 41.0 and 2.75, respectively), and converting the base of the exponential term to base 10. The resulting equation can be represented as follows:

$$G(\alpha, v) \quad = \quad [250.1 + 299.3|\log_{10}(v/3)|^3]v\alpha^2 10^{-5.5\alpha(v+2)/45.9}. \tag{6}$$

This equation is plotted in Figure 2 for a number of velocities. From this we can observe that the effect of velocity on the CSF is to push the curve further towards the y-axis for higher velocities, i.e. we can see less high detail with increasing velocity.

## 3.2 Incorporating Eccentricity into the Model

Contrast sensitivity declines with increasing eccentricity. However the shape of the spatiotemporal surface is consistent across the visual field (Virsu *et al.*, 1982; Koenderink *et al.*, 1978; Kelly, 1984). This would lead us to believe that we can predict the contrast sensitivity for any region of the retina by simply scaling the foveal response with a factor based upon eccentricity.

Rovamo and Virsu (1979) confirmed this when they showed that visual acuity can be accurately predicted for any eccentricity by applying a constant scaling factor, referred to as the *cortical magnification factor* (M), first introduced by Daniel and Whitteridge (1961). Therefore, in order to incorporate eccentricity into our model of spatiotemporal contrast sensitivity, we simply need to apply this cortical magnification factor to Equation 6.

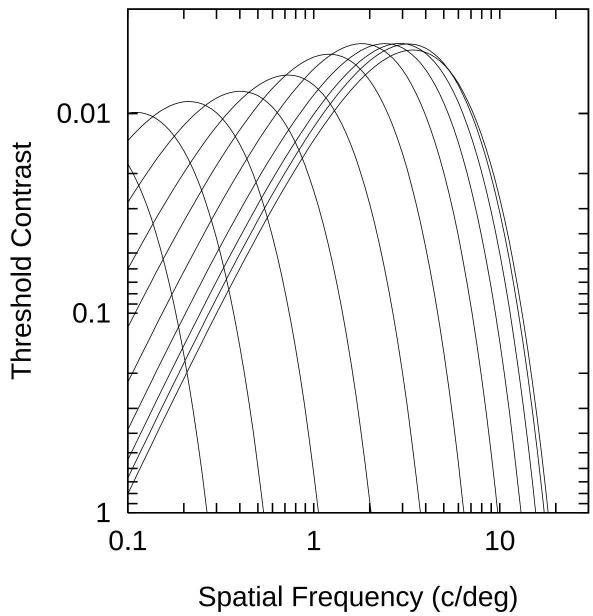



Figure 2: *(a) Contrast sensitivity functions for velocities of 0.125, 0.25, 0.5, 1, 2, 4, 8, 16, 32, 64, and 128 deg/s (from right to left); calculated using Equation 6.*

The eye's peripheral sensitivity is not circular symmetric (e.g. Regan and Beverley, 1983). For example, there are marked asymmetries between the nasal and temporal retina beyond 20 deg (Sutter and Tran, 1991). Taking this into consideration, Rovamo and Virsu produced four equations to characterise M for each principal half-meridian of the retina. These are replicated below, and plotted in Figure 3(a).

$$\text{Nasal:} \quad M_N = M_0/(1 + 0.33E + 0.00007E^3), \quad 0 \leq E \leq 60 \text{ deg.} \tag{7}$$

$$\text{Superior:} \quad M_S = M_0/(1 + 0.42E + 0.00012E^3), \quad 0 \leq E \leq 45 \text{ deg.} \tag{8}$$

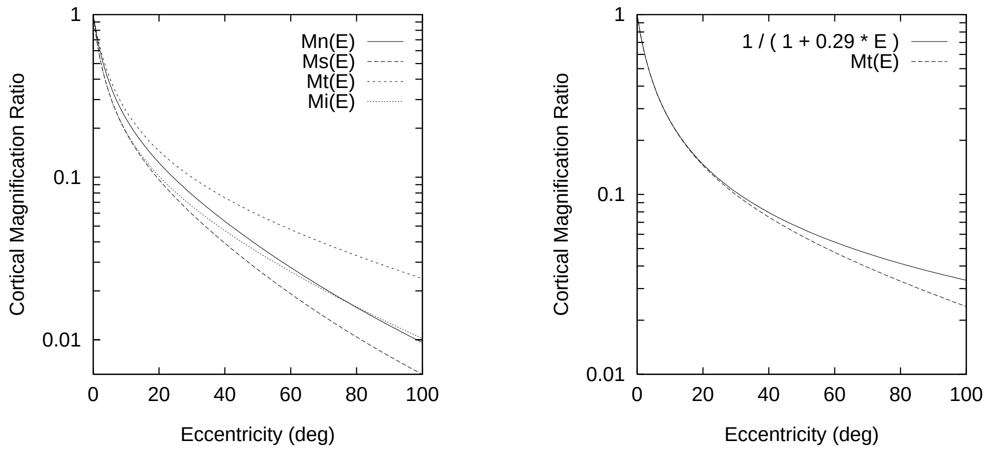
$$\text{Temporal:} \quad M_T = M_0/(1 + 0.29E + 0.000012E^3), \quad 0 \leq E \leq 80 \text{ deg.} \tag{9}$$

$$\text{Inferior:} \quad M_I = M_0/(1 + 0.42E + 0.000055E^3), \quad 0 \leq E \leq 60 \text{ deg.} \tag{10}$$

Where $M_0$ is the value of magnification for the most central point in the fovea; which we can simply instantiate as $M_0 = 1$. To simplify the above relationship, we could exclusively use the most sensitive region's M, with the knowledge that the other regions will not exceed this sensitivity; i.e. Equation 9 ($M_T$).

Also, it would be reasonable to ignore the cubic term in Equation 9. This only becomes significant at large eccentricities; and even when $E = 100$ deg, there would only be an error of roughly 1%. This simplification was adopted by Watson (1983), Kelly (1984), and Tyler (1985) in their respective models, among others. We can therefore define the cortical magnification factor for our purposes as:

$$M = M_0/(1 + 0.29E). \tag{11}$$




**(a)** **(b)**

Figure 3: *(a) A graph of Equations 7–10, which define the cortical magnification factor, M, for each cardinal half-meridian of the retina. (b) A comparison of Equation 9 with the cubic term included (lower curve) and with the cubic term ignored (upper curve).*

We can subsequently incorporate this equation into our model for contrast sensitivity as follows:

$$\begin{aligned} H(\alpha, v, E) &= G(\alpha, v) \times M, \\ &= G(\alpha, v)/(1 + 0.29E). \end{aligned} \tag{12}$$

## 3.3 Deriving Visual Acuity from the Model

In order to describe the visual acuity of an observer in terms of spatial frequency, we wish to calculate the upper point where the CSF intersects the x-axis. This point is defined as a contrast sensitivity of one, i.e. $H(\alpha, v, E) = 1$. The solution to calculating the highest detail which a user can perceive for any velocity and eccentricity can therefore be written as:

$$H(\alpha, v, E) - 1 = 0. \tag{13}$$

We then need to solve this equation in terms of $\alpha$ in order to discover the spatial frequency at which $H(\alpha, v, E) = 1$, taking the highest root as the threshold size. An analytical solution to this problem would be overtly complicated; requiring the computation of Lambert's W function (Corless *et al.*, 1993), or equivalent, to resolve the combination $\alpha^2 \exp(\alpha)$ which arises. A more tractable solution can be found by using an iterative method such as an interval halving algorithm or a Newton-Raphson technique.

# 4 Evaluating the Model

Equation 13 provides us with a method to estimate the visual acuity (c/deg) of a human observer to a stimulus at any arbitrary eccentricity ($E$ deg), moving at a particular velocity ($v > 0.1$ deg/s). We now wish to evaluate this model to see how well it performs on potential computer graphics imagery. Accordingly, this section describes two psychophysical experiments to investigate the accuracy of our model in the task of predicting a user's visual acuity under various conditions. We will evaluate the two facets of our perceptual model independently so that we may gain some insight into the efficiency of each individual component. We will therefore begin by looking at how well our model predicts the visibility of stimuli based upon their eccentricity, and then proceed to investigate the same for varying velocity.

## 4.1 Eccentricity Effects on Visual Acuity

### 4.1.1 Objective

This experiment was devised to assess the effective threshold eccentricity for a number of 2D, aperiodic stimuli at a fixed contrast and spatial frequency. This was performed in order to establish how well our perceptual model can predict the user's ability to resolve detail in their peripheral field.

### 4.1.2 Overview

The user was presented with a number of simple stimuli at various points in their peripheral field. The experiment was devised to locate the user's threshold eccentricity for a stimulus of fixed size and contrast. This was then compared with the predicted threshold from our perceptual model.

A standard *2 alternative forced choice* (2AFC) methodology was adopted, using a *staircase* technique. 2AFC means that the user is presented with two intervals, with the test stimulus appearing in only one of these. They must then state which interval they believe the test stimulus occured in; guessing if necessary (Lamming, 1991). A staircase technique is a procedure that adaptively selects new stimulus difficulties (i.e. eccentricities in this experiment) based upon the subject's previous responses; eventually converging around the threshold value (Wetherill and Levitt, 1965).

### 4.1.3 Method

*Stimuli*. The stimuli were displayed at full contrast (a white stimulus on a 2 deg black patch) on a mid-grey background. The display consisted of two black patches, one to the left and one to the right of the crosshair (both patches were equidistant from the crosshair). The stimulus was displayed randomly in either the left or the right interval and retained the same contrast and spatial frequency throughout the experiment; only varying in the eccentricity at which it was displayed to the user. Figure 4 illustrates the stimuli used for this experiment.

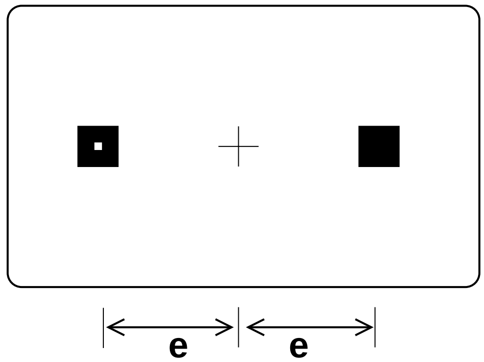



Figure 4: *The display used for the eccentricity 2AFC test. The stimulus was presented at eccentricity* e, *in either the left or the right patch (shown here on the left).*

The experiment was run for a number of different sized stimuli (1 and 2 pixels square) at a number of different viewing distances (10, 20, 30, 40, 50, and 60 cm). Overall, seven different spatial frequencies were analysed: 2.52, 5.04, 7.11, 9.50, 12.05, 14.68, and 17.36 c/deg. The experiment terminated once the staircase algorithm had converged to a threshold eccentricity.

*Procedure*. Three subjects, with normal or corrected-to-normal vision, performed each experiment twice (i.e. a total of 14 experiments were undertaken by each subject). All subjects were unpaid male postgraduate students in the Department of Computer Science at the University of Edinburgh.

Each subject fixated on a crosshair displayed at eye level on the computer monitor. Eye movements were not monitored, however a chin rest was used in order to restrict head movement and to preserve the viewing distance. The subject was asked to chose the correct interval by pressing one of two buttons on the keyboard. They could abort any observation with another button if desired and this data would not be included in the final result. No feedback was given to subjects on their performance during the course of the experiments.

Subjects were allowed a number of test runs beforehand in order to acquaint themselves with the experimental technique. The final threshold figures were found by averaging each pair of results from all subjects.

### 4.1.4 Results

The results from this experiment are presented in Figure 5. In order to calculate the predicted threshold response, we used our cortical magnification factor, defined by Equation 11. This can be written in terms of spatial frequency ($\alpha$) as:

$$E = (M_0/\alpha - 1)/0.29, \qquad (14)$$

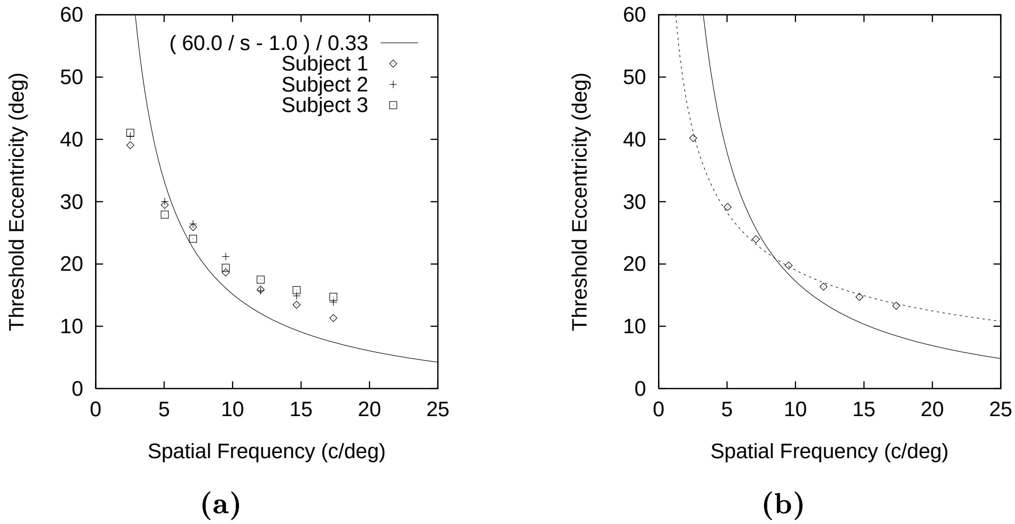



Figure 5: *Results of the eccentricity experiment. (a) presents the individual responses of each of the three subjects, while (b) presents the averaged response for all subjects (the data points) and the best fit curve to this average response (the dotted line: Equation 15). The continuous line in (a) and (b) represents the predicted response from our model, given by Equation 14.*

where $M_0 = 60$ c/deg: the highest spatial frequency which can be resolved at the fovea (Campbell and Gubisch, 1966).

The experiment produced results which were in the correct order of magnitude, but somewhat displaced from our predicted curve. A best fit curve was therefore calculated which represented the data more closely. This curve is defined in Equation 15 below, and plotted in Figure 5(b).

$$E = (21.2/\alpha^{0.5} - 1)/0.3. \tag{15}$$

We can use this curve to formulate a new model for our cortical magnification factor by first rewriting Equation 15 in terms of spatial frequency, e.g.

$$\begin{aligned} \alpha &= (21.2/(0.3E + 1))^2 \\ &= 449.44/(0.3E + 1)^2, \end{aligned} \tag{16}$$

and then normalising this so that a value of 1.0 is returned at the fovea and values tending towards 0.0 are returned for highly eccentric locations. We have already reported that the highest spatial frequency visible at the fovea is 60 c/deg. Using Equation 15, we find that our best fit curve does not go below this spatial frequency until an eccentricity of 5.79 deg is exceeded. We can therefore present our new equation for cortical magnification as follows (this equation is plotted in Figure 6):

$$M = \begin{cases} 1.0, & \text{when } E \leq 5.79 \\ 449.44/((0.3E+1)^2 \times 60) & \\ \quad = 7.49/(0.3E+1)^2, & \text{when } E > 5.79. \end{cases} \quad (17)$$

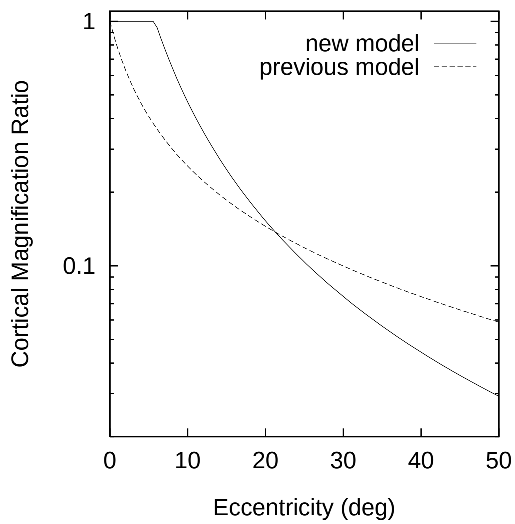



Figure 6: *Comparison between our new and previous definitions of the cortical magnification factor, $M$. The solid curve represents our new empirically derived model (Equation 17), whilst the broken curve represents our previous definition (Equation 11).*

#### 4.1.5 Discussion of the Eccentricity Experiment

The results from this experiment are very encouraging. They display a clear and smooth decline in spatial frequency sensitivity with increasing eccentricity. The results for all subjects lie consistently within 2 deg of the average result, and mostly within 1 deg. Given the inherent variability of individuals' vision systems, and taking into consideration any experimental error, this is a very good result. However, it is apparent that the empirical data, although in the correct order of magnitude, does not exactly match the theoretical threshold. This could be due to inaccuracies in our theoretical model (Equation 11), however, the most likely reason for the discrepancy is that, being based solely on retinal ganglion cell distributions, Rovamo and Virsu's (1979) model may not encapsulate the total processing performed by the entire visual system.

We therefore produced a best fit curve (Equation 15) which better models the observers' decline of peripheral sensitivity, and subsequently formulated a new model of cortical magnification for our application (Equation 17).

### 4.2 Velocity Effects on Visual Acuity

#### 4.2.1 Objective

The second psychophysical experiment was devised to assess various subjects' threshold velocity for a number of 2D, aperiodic stimuli at a fixed contrast and spatial frequency. This was performed in order to establish how well our perceptual model can predict the user's ability to resolve detail based upon the velocity of a stimulus.

#### 4.2.2 Method

Stimuli. Two black 2 deg patches were animated vertically past the observer's viewpoint (at 72 Hz) with a constant angular velocity; one to the left of the crosshair and one to the right. A white (full contrast) stimulus was randomly displayed in either the left or right interval during each trial and the observer had to chose which interval they thought the stimulus had appeared in. The stimuli were always presented at a horizontal angular distance of 2.5 deg from the crosshair in order to minimise the effect of eccentricity on detection (note that our results in Figure 6 show that all potential stimuli would be visible under static conditions within 0 to 5.79 deg). Figure 7 illustrates the display used for the experiment.

A 2AFC staircase method was adopted as before. Six different spatial frequencies were analysed: 5.04, 7.11, 9.50, 12.05, 14.68, and 17.36 c/deg.

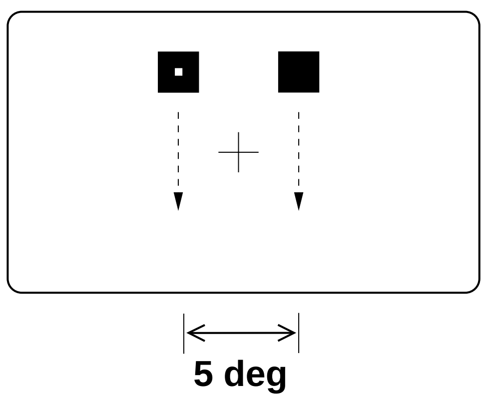
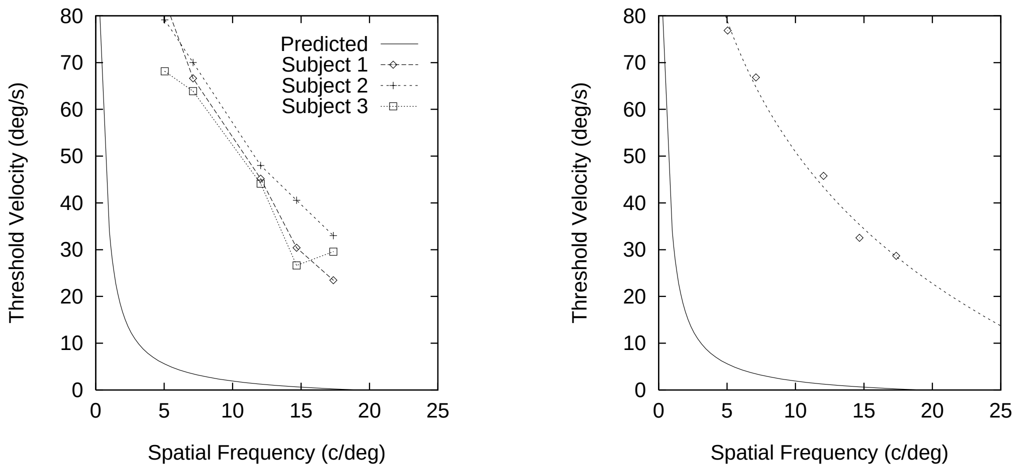



Figure 7: *The display used for the velocity 2AFC test. Two patches moved past the observer's fixation point at a constant angular velocity, with the stimulus being present in either the left or the right patch (shown here on the left).*

Procedure. The same three subjects who performed the previous experiment were solicited for this experiment. Each subject fixated upon a crosshair, positioned at eye level, with their head movement constrained by a chin rest. Once again, subjects were allowed a number of test runs beforehand in order to acquaint themselves with the experimental technique. No feedback was given.

The experiment was performed twice for each spatial frequency (i.e. each subject performed 12 experiments in total). The final threshold figures were found by averaging each pair of results from all subjects.

### 4.2.3 Results




**(a)** **(b)**

Figure 8: *Results of the velocity experiment. (a) presents the individual responses of each of the three subjects, while (b) presents the averaged response for all subjects (the data points) and the best fit curve to this average response (the dotted line: Equation 18). The continuous line in (a) and (b) represents the predicted response from our model.*

The results of the velocity experiment are presented in Figure 8. In order to calculate the predicted threshold response, we used our spatiotemporal contrast sensitivity model (Equation 6) and found the upper root of the relation, $G(\alpha, v) - 1 = 0$, using an interval halving algorithm.

As can be observed from the data in Figure 8, the results which were obtained are substantially deviant from our theoretical model. A best fit curve was therefore calculated for the experimental data (using a computer graphing package). An exponential curve fit was used in order to retain as much as possible the general characteristics of Kelly's (1975) abstract model (see Section 3). The resulting curve is presented in Equation 18 below, and plotted in Figure 8(b).

$$v = 119.353 \times 10^{-0.036\alpha}. \tag{18}$$

We can use this curve to help us formulate a new spatiotemporal threshold model. This requires that we find the corresponding form of Equation 18 in terms of spatial frequency, $\alpha$.

That is,

$$\begin{aligned} \alpha &= \log_{10}(v/119.353)/{-0.036} \\ &= (\log_{10}(v) - \log_{10}(119.353))/{-0.036} \\ &= -27.78\log_{10}(v) + 57.69. \end{aligned} \tag{19}$$

Once again we can note that this equation only goes below 60 c/deg when $v > 0.825$ deg/s. We can therefore present the final definition for our spatiotemporal threshold model (for $v \geq 0$ deg/s) as follows. This equation is plotted in Figure 9 along with the previous model for comparison.

$$\alpha = \begin{cases} 60.0, & \text{when } v \leq 0.825 \\ -27.78\log_{10}(v) + 57.69, & \text{when } v > 0.825. \end{cases} \tag{20}$$

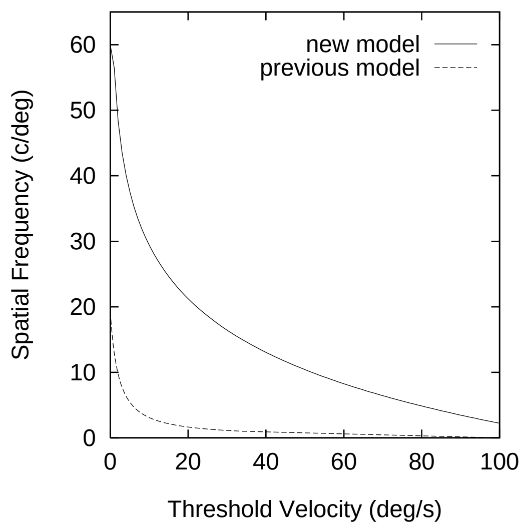



Figure 9: *Comparison between our new and previous definitions for the spatiotemporal threshold surface. The solid curve represents our new empirically derived model (Equation 20), whilst the broken curve represents our previous definition (using Equation 6).*

#### 4.2.4 Discussion of the Velocity Experiment

The results from this experiment are not as smooth as those obtained from the eccentricity experiment, or as consistent with the theoretical response. The most probable cause for the observed discrepancy is due to the fact that we are using very localised, aperiodic stimuli (in terms of the field of view occupied), whereas vision scientists normally deal with extended stimuli which fill a large proportion of the FOV. Also, the experiment was more complicated, both for the author to devise, and for the subjects to perform. In the first instance, it would be impossible to completely isolate the effect of eccentricity using a spatial 2AFC test because the stimulus must be presented at some displacement left or right from the fixation point; also, because the stimulus is moving, its eccentricity is constantly changing. Secondly, the experiment is more difficult for the subjects to perform accurately because it is an instinctive reflex to fixate upon and track moving objects, thus altering their effective angular velocity.

We therefore produced a best fit curve (Equation 18) which models the observed decline of temporal sensitivity more closely; and subsequently, we formulated a new model of spatio-temporal threshold for our application (Equation 20). It may be noted that this equation, as well as providing a more accurate and practical model, is also significantly less complex to compute than our original model.

# 5 Discussion

Using the results from our eccentricity and velocity psychophysical studies, we can now present a re-implementation of our visual acuity model. The following equation provides a value for the highest visible spatial frequency (c/deg) that a standard observer can resolve under any eccentricity ($E \geq 0$ deg) and velocity ($v \geq 0$ deg/s):

$$H(v,E) = G(v) \times M(E), \tag{21}$$

where,

$$G(v) = \begin{cases} 60.0, & \text{when } v \leq 0.825 \\ -27.78\log_{10}(v) + 57.69, & \text{when } v > 0.825 \end{cases} \tag{22}$$

$$M(E) = \begin{cases} 1.0, & \text{when } E \leq 5.79 \\ 7.49/(0.3E+1)^2, & \text{when } E > 5.79. \end{cases} \tag{23}$$

Our formulation of this new model for visual acuity is valid because we are concerned with sufficiently different stimuli from those which normally preoccupy vision scientists. For example, we are concerned with local, aperiodic, non-harmonic stimuli whilst vision scientists normally deal with contrast gratings which are harmonic, periodic, and extend over a large field of view. We have therefore tailored our model to the genus of stimuli commonly found in computer-generated imagery.

It is worth noting that this model describes our real-world ability to resolve detail. When viewing a computer display our spatial resolution will be affected by the resolution of that display. This can be easily incorporated into our model by simply thresholding (taking the minimum value) between the result of $H(v,E)$ and the highest spatial frequency which the display device can produce.

One factor which is notably absent from our model is the ambient illumination level. We know that our ability to perceive detail varies with respect to the background illumination; with our resolving ability degrading in darker surrounds (Kelly, 1975). However, Kelly's (1979) model was formulated for high background illumination. This is therefore a worst-case model, and so implicitly handles the case of low background illumination. We therefore do not need to consider the effects of ambient illumination in our visual acuity model.

## 6 Conclusions

The product of this paper has been the development of a computational model for visual acuity which is simple enough to be implemented in real-time. This has been done so that we may know the smallest size of detail (highest spatial frequency) that a user can perceive under any circumstances. Together with the previous work in Reddy (1996), we now have a means to represent the perceived size of arbitrary features in a computer-generated image, and to decide whether a user should be able to resolve these features.

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