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Standing still with inertial navigation

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***Abstract*—The possibility to detect complete standstill sets foot-mounted inertial navigation aside from other heuristic pedestrian dead reckoning systems. However, traditional zero-velocity-updates (ZUPTs) does not ensure that the system is actually static but a drift will occur. To eliminate the drift we suggest using a modified mechanization which locks certain states under certain conditions. Experimental data is used to demonstrate the effectiveness of the approach.**

I. Introduction

Just like we spend most of our time indoor, we spend most of our time being still. Consequently, an important attribute of any pedestrian dead reckoning system is that it gives good performance for standstill. Due to its mounting point and implicit dynamic assumptions, ZUPT-aided foot-mounted inertial navigation works well for such situations, providing an attribute that sets it aside from other heuristic (step counting) pedestrian dead reckoning systems. However, the zero-velocity updates, as they are typically implemented, will not make an inertial navigation system stand still, but a drift will occur. Naively, this may be avoided by stopping the integration in the inertial navigation simultaneously with the ZUPTs. However, we argue that due to modelling errors this is not advisable. Instead, in this short paper we suggest an alternative mechanization which locks certain states during certain (standstill) conditions. Experimental data is used to demonstrate that this stand-still mechanization improves performance for situations where a user is standing still, while not influencing the behaviour of the system for normal gait.

II. Foot-mounted inertial navigation

Foot-mounted ZUPT-aided inertial navigation consists of foot-mounted inertial sensors, inertial mechanization equations, a zero-velocity detector, and a complementary Kalman filter based on a deviation (error) model to fuse the information. In the simplest form the mechanization equations are

$$\begin{bmatrix}\mathbf{p}_k\\ \mathbf{v}_k\\ \mathbf{q}_k\end{bmatrix}=\begin{bmatrix}\mathbf{p}_{k-1}+\mathbf{v}_{k-1}dt\\ \mathbf{v}_{k-1}+(\mathbf{q}_{k-1}\mathbf{f}_k\mathbf{q}^{\star}_{k-1}-\mathbf{g})dt\\ \mathbf{\Omega}(\boldsymbol{\omega}_{\boldsymbol{k}}dt)\mathbf{q}_{k-1}\end{bmatrix} \tag{1}$$

where k is a time index, dt is the time difference between measurement instances, $\mathbf{p}_k$ is the position, $\mathbf{v}_k$ is the velocity, $\mathbf{f}_k$ is the specific force, $\mathbf{g}=[0\,0\,g]^\top$ is the gravity, $[\cdot]^\top$ denotes the transpose operation, and $\boldsymbol{\omega}_k$ is the angular rate (all in $\mathbb{R}^3$). Further, $\mathbf{q}_k$ is the quaternion describing the orientation of the system, the triple product $\mathbf{q}_{k-1}\mathbf{f}_k\mathbf{q}^{\star}_{k-1}$ denotes the rotation of $\mathbf{f}_k$ by $\mathbf{q}_k$, and $\mathbf{\Omega}(\cdot)$ is the quaternion update matrix. For analytical convenience we will interchangeably represent the orientation $\mathbf{q}_k$ with the equivalent Euler angles $\boldsymbol{\theta}_k=[\phi_k\ \theta_k\ \psi_k]^\top$ (roll, pitch, yaw) or the rotation matrix $\mathbf{R}_k$.

The mechanization equations together with measurements of the specific force $\tilde{\mathbf{f}}_k$ and the angular rates $\tilde{\boldsymbol{\omega}}_k$, provided by the inertial sensors, are used to propagate position $\hat{\mathbf{p}}_k$, velocity $\hat{\mathbf{v}}_k$, and orientation $\hat{\mathbf{q}}_k$ state estimates. Unfortunately, due to its integrative nature, small errors in $\tilde{\mathbf{f}}_k$ and $\tilde{\boldsymbol{\omega}}_k$ accumulate, giving rapidly growing state estimation errors. Fortunately, these errors can be modeled and estimated with ZUPTs. A first-order deviation (error) model of (1) is given by

$$\begin{bmatrix}\delta\mathbf{p}_k\\ \delta\mathbf{v}_k\\ \delta\boldsymbol{\theta}_k\end{bmatrix}=\begin{bmatrix}\mathbf{I} & \mathbf{I}dt & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & [\mathbf{q}_{k-1}\mathbf{f}_k\mathbf{q}^{\star}_{k-1}]_\times dt\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\delta\mathbf{p}_{k-1}\\ \delta\mathbf{v}_{k-1}\\ \delta\boldsymbol{\theta}_{k-1}\end{bmatrix} \tag{2}$$

where $\delta(\cdot)_k$ are the error states, $\mathbf{I}$ and $\mathbf{0}$ are 3×3 identity and zero matrices, and $[\cdot]_\times$ is the cross-product matrix. Together with statistical models for the errors in $\tilde{\mathbf{f}}_k$ and $\tilde{\boldsymbol{\omega}}_k$, (2) is used to propagate statistics (covariances) of the error states. To estimate the error states, stationary time instances are detected based on the condition $Z(\{\tilde{\mathbf{f}}_\kappa,\tilde{\boldsymbol{\omega}}_\kappa\}_{W_k})<\gamma_{\text{Z}}$, where $Z(\cdot)$ is some zero-velocity test statistic, $\{\tilde{\mathbf{f}}_\kappa,\tilde{\boldsymbol{\omega}}_\kappa\}_{W_k}$ is the inertial measurements over some time window W_k, and γ_{Z} is a zero-velocity detection threshold. See [1] for further details. The implied zero-velocities are used as pseudo-measurements

$$\tilde{\mathbf{y}}_k=\hat{\mathbf{v}}_k\ \forall k: Z(\{\tilde{\mathbf{f}}_\kappa,\tilde{\boldsymbol{\omega}}_\kappa\}_{W_k})<\gamma_{\text{Z}} \tag{3}$$

which are modeled in terms of the error states as

$$\tilde{\mathbf{y}}_k=\mathbf{H}\begin{bmatrix}\delta\mathbf{p}_k & \delta\mathbf{v}_k & \delta\boldsymbol{\theta}_k\end{bmatrix}^\top+\mathbf{n}_k \tag{4}$$

where $\mathbf{H}=[\mathbf{0}\ \mathbf{I}\ \mathbf{0}]$ is the measurement matrix and $\mathbf{n}_k$ is a measurement noise, i.e. $\tilde{\mathbf{y}}_k=\delta\mathbf{v}_k+\mathbf{n}_k$. Given the error model (2) and the measurements model (4), the measurements (3) can be used to estimate the error states with a complementary Kalman type of filter. See [2,3] for further details.

III. Inertial navigation during standstill

During standstill, obviously ZUPTs are in effect and velocity and roll and pitch are observable [4]. However, this does not imply that the system is standing still, neither in a physical sense nor in a state estimation sense. The constraint that the ZUPTs add is that the system has zero-mean velocity with a certain distribution, i.e. $\hat{\mathbf{v}}_k-\delta\hat{\mathbf{v}}_k=\mathbf{n}_k$. Unfortunately, measurement noise will enter the system in both (1) and (2) (since $\mathbf{f}_k$ and $\boldsymbol{\omega}_k$ are replaced with their measured values) causing the system to drift even if consecutive ZUPTs are applied. Consequently, we would like to make the system stand still to avoid this. However, in practice it has been shown that the threshold γ_{Z}, giving the best performance, is far above the statistic's noise floor [5], and the system will not necessarily be perfectly stationary when ZUPTs are applied [6,7].

To remedy the drift during standstill, the system has to be locked somehow. However, as seen in the discussion above, this has to be done with care. First of all, as seen in [6], a different condition than $Z(\{\tilde{\mathbf{f}}_\kappa, \tilde{\boldsymbol{\omega}}_\kappa\}_{W_k}) < \gamma_{\text{Z}}$ has to be used to lock the system. The same detection framework may be used but with a lower threshold, a larger window, and explicit modelling of the gyro bias. Consequently, we will assume that a corresponding stationarity detector is employed with a threshold γ_{S} set just above the statistic's noise floor. If the condition holds, the inertial navigation is to be locked to mitigate drift. However, to avoid numerical problems and to avoid introducing unnecessary modelling errors, we only wish to introduce a minimum of locking. We are concerned about the drift in the position states and in the heading. The remaining states are observable. Therefore, only the position and the heading should be locked. Locking the position is trivial while locking only the heading is not. The problem is that we would typically not use a zero-order hold assumption of the rotation and there is no clear-cut heading change separable from $\boldsymbol{\Omega}(\boldsymbol{\omega}_{\boldsymbol{k}} dt)\mathbf{q}_{k-1}$ in (1). However, since the angular rates can be assumed small when we want to apply a heading lock, this can be achieved by subtracting the angular rate component orthogonal to the horizontal plane, which primarily affects the heading. The angular rates in the navigation coordinate system are $\mathbf{R}_{k-1}\boldsymbol{\omega}_k$ and consequently the angular rates with the horizontal plane component subtracted are $\mathbf{R}_{k-1}\boldsymbol{\omega}_k - \text{diag}([001])\mathbf{R}_{k-1}\boldsymbol{\omega}_k$, where $\text{diag}([001])$ is the diagonal matrix with $[001]$ on the diagonal. Transforming back to the body coordinate system (multiplying with $(\mathbf{R}_{k-1})^\top$ from the left) gives the desired quantities

$$\boldsymbol{\omega}_k - (\mathbf{R}_{k-1})^\top \text{diag}([001])\mathbf{R}_{k-1}\boldsymbol{\omega}_k.$$

Consequently, a standstill mechanization with locks on the position and heading is given by

$$\begin{bmatrix}\mathbf{p}_k\\ \mathbf{v}_k\\ \mathbf{q}_k\end{bmatrix} = \begin{bmatrix}\mathbf{p}_{k-1}\\ \mathbf{v}_{k-1} + (\mathbf{q}_{k-1}\mathbf{f}_k\mathbf{q}^\star_{k-1} - \mathbf{g})dt\\ \boldsymbol{\Omega}((\boldsymbol{\omega}_k - (\mathbf{R}_{k-1})^\top \text{diag}([001])\mathbf{R}_{k-1}\boldsymbol{\omega}_k)dt)\mathbf{q}_{k-1}\end{bmatrix} \quad (5)$$

and the corresponding deviation model is

$$\begin{bmatrix}\delta\mathbf{p}_k\\ \delta\mathbf{v}_k\\ \delta\boldsymbol{\theta}_k\end{bmatrix} = \begin{bmatrix}\mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{I} & [\mathbf{q}_{k-1}\mathbf{f}_k\mathbf{q}^\star_{k-1}]_\times dt\\ \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix} \begin{bmatrix}\delta\mathbf{p}_{k-1}\\ \delta\mathbf{v}_{k-1}\\ \delta\boldsymbol{\theta}_{k-1}\end{bmatrix}. \quad (6)$$

During complete standstill, i.e. when the second detector is true ((3) will apply then as well), instead of (1) and (2), (5) and (6) should be used and the heading process noise covariance set to zero. Thereby the desirable locking effect of position and heading is attained.

IV. Experiment

The standstill mechanization (5) and (6) has been implemented in the OpenShoe platform [8]. The implementation can be found at **www.openshoe.org**. To demonstrate the effectiveness of the mechanization we show the dead reckoning behaviour for a dataset where the system is truly standing still and for a dataset of normal gait. The results are seen in

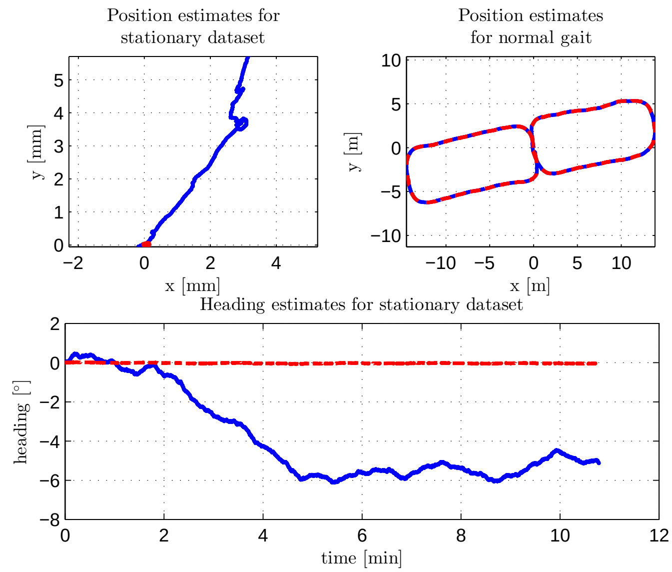

Fig. 1. State estimates with stand-still mechanization when applicable (dashed red) and without (solid blue) for an 11 minutes stationary period (upper left and bottom plots) and for a walk in a figure eight (upper right plot).

Fig. 1. For the static scenario (upper left and lower plots), the locking mechanization keeps the estimates stable while they drift without it. For the normal gait (dynamic) scenario (upper left plot), the estimated trajectories are seen to overlap and the alternating stand-still mechanization does not affect the inertial navigation (as expected).

V. Conclusions

Zero-velocity updates of inertial navigation does not ensure that an inertial navigation system is actually static but a drift will occur. This can preferably be remedied by applying the suggested alternative mechanization which locks the position and heading under certain standstill conditions. The pedestrian dead reckoning performance during standstill has been shown to improve with the suggested mechanization while not affecting the behaviour for normal gait.

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