A General Method for Synthesis of Constant-Amplitude Ideal Periodic Autocorrelation Phase Manipulated Signals

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Introduction

The constant-amplitude ideal periodic autocorrelation phase manipulated signals (CAIPAPMSs) find many applications in the present radio-communication systems (RCSs) due to their positive features. As a result, a large set of methods for synthesis of PM signals with ideal periodic autocorrelation has been proposed and practically adopted. Accounting this situation in the paper a general method for synthesis of CAIPAPMS is substantiated. It describes all the known methods for synthesis of CAIPAPMSs and opens new directions for research in this theoretical area.

Transformations, Preserving the Autocorrelation Properties of Phase Manipulated Signals

The performance of the RCSs is negatively affected by: the natural noises, the self-interferences between the copies of a radio-signal, passed different ways among the transmitter and the receiver, the mutual interferences between radio-electronic devices, working simultaneously, as well as by the active jamming. One of the signal types, providing the greatest resistance to the listed negative factors, are the constant-amplitude PM signals, which periodic autocorrelation functions (PACFs) have perfect (or ideal) form, resembling a delta pulse.

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These PM signals are sequences of N elementary phase pulses with equivalent duration τ_{ch}, named symbols or chips. As a result, they can be described by the following mathematical model:

$$\{s(i)\}_{i=0}^{N-1} = \{s(0), s(1), \dots, s(N-1)\}. \qquad (1)$$

Here $s(0), s(1), \dots, s(N-1)$ are complex numbers (which will be named samples in sequel) with (initial) phases φ_i and unit absolute values $|s(i)| = 1$:

$$s(i) = e^{j\varphi_i}, j = \sqrt{-1}, i = 0, 1, \dots, N-1. \qquad (2)$$

They present the complex envelopes of the symbols (chips).

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The PACF of a constant-amplitude PM signal (1) resembles a delta pulse (has a thumbtack form), if:

$$Q_{ss}(r) = \sum_{i=0}^{N-1} s(i)s^*\langle i+r\rangle_N = \begin{cases} N, r = 0, \\ 0, r \neq 0. \end{cases} \qquad (3)$$

In (3) $Q_{ss}(r)$ is the lobe of the PACF for the time shift $r\tau_{ch}, r = 0, 1, \ldots, N-1$.

For the next analysis it is necessary to be recalled that one PM signal can be transformed in another PM signal with equivalent autocorrelation properties. These equivalent transformations (ET) are:

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ET 1) Complex conjugation of the PM signal samples.

ET 2) Mirror reverse of the PM signal samples.

ET 3) Cyclic shift of the PM signal samples at $l, l = 1, 2, \dots, N-1$ positions.

ET 4) Multiplication of the PM signal samples by arbitrary complex number $\delta, |\delta| = 1$ with unit absolute value.

ET 5) Multiplication of the PM signal samples by a “progressive” factor $\epsilon(i) = e^{j\frac{2\pi m}{N}i}, (m, N) = 1, 1 \le m \le N-1$.

ET 6) Decimation of the PM signal samples.

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As all the samples, building a CAIPAPMS, have unit absolute value, (3) can be presented in the form

$$Q_{ss}(r) = \sum_{i=0}^{N-1} \frac{s(i)}{s\langle i+r\rangle_N} = \begin{cases} N, r = 0, \\ 0, r \neq 0. \end{cases} \tag{12}$$

After applying the substitution

$$z(i) = \frac{s(i)}{s\langle i+1\rangle_N}, \quad i = 0, 1, \dots, N-1, \tag{13}$$

(12) transforms to:

$$Q_{ss}(r) = \sum_{i=0}^{N-1} \left[\prod_{l=0}^{r-1} z\langle i+l\rangle_N\right] = 0, r \neq 0. \tag{14}$$

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Another consequence from (13) is the relation:

$$z(i)z\langle i+1\rangle_N \ldots z\langle i+N-1\rangle_N = \prod_{l=0}^{N-1} z\langle i+l\rangle_N = 1. \quad (15)$$

The combination of (14) for $r = 1, 2, \ldots, N-1$ and (15) produces the following system of equations:

$$\left|\begin{array}{l} z(0)+z(1)+\cdots+z(N-1)=0 \\ z(0)z(1)+z(1)z(2)+\cdots+z(N-1)z(0)=0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \sum_{i=0}^{N-1}\left[\prod_{l=0}^{k-1} z\langle i+l\rangle_N\right]=0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ z(0)z(1)\ldots z(N-1)=1 \end{array}\right. \quad (16)$$

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The solutions of the system of equations (16) are named cyclic N-roots, as they are cyclic equivalent. The system of equations (16) is invariant relatively to the complex conjugation and it can be presented also in the form:

$$\left|\begin{array}{c} z(0)+z(1)+\cdots+z(N-1)=0 \\ z(0)z(1)+z(1)z(2)+\cdots+z(N-1)z(0)=0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \\ z^*(0)z^*(1)+\cdots+z^*(N-1)z^*(0)=0 \\ z^*(0)+z^*(1)+\cdots+z^*(N-1)=0 \\ z(0)z(1)\ldots z(N-1)=1 \end{array}\right. \qquad (17)$$

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On the base of (17) it is proved that the cyclic N-roots are all the roots of a polynomial (named in the paper characteristic polynomial), which is described by the following proposition:

***Proposition*:** The characteristic polynomial, defined by (16), is:

$$H(x) = x^N + h_{N-2}x^{N-2} + \cdots + h_2x^2 + h_0, \quad (43)$$

where

$$h_k = (-1)^N h_{N-k}^*, \; k = 2, 3, \ldots, N-2 \text{ and } h_0 = (-1)^N.$$

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As a result, in the paper the characteristic polynomials of:

- all the possible CAIPAPMSs with lengths $N = 2, 3, 4,$

-all the known CAIPAPMSs, which signal alphabets are strongly restricted to be a subset of a complete set of roots of unity,

are described.

Conclusion

In the paper a new general method for synthesis of CAIPAPMSs is substantiated. On its base it is possible all methods for synthesis of CAIPAPMSs to be analytically described, classified and applied for development of many types of intelligent communication devices, exploited in smart vehicles.

Besides, the developed new method can be further elaborated for synthesis of constant-amplitude PM signals with arbitrary prescribed (desired) PACF.