

# Music from Irrational Numbers

## Creating a piece of music from $\sqrt{2}$

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### Abstract

In this project I attempted to create a piece of music using an irrational number. I chose a scale and assigned the digits 0-9 to that scale and then used the digits from the irrational number to create a string of notes. Then, using a computer program called MuseScore, I inputted these onto some sheet music and added multiple layers to create a full piece of music.

## Introduction

As a musician I have always found that music can be beautiful. Some of the best music is just so because of how the artist/composer takes deliberate care in choosing each individual note and chord and can potentially spend days putting it all together until it's perfect. Knowing this, in my project I decided to introduce an element of randomness to see if it were possible to create music this way. My main objectives were:

- Choose a scale to use
- Choose an irrational number to convert to notes
- Experiment on MuseScore to create a piece of music

## Irrational Numbers

An irrational number is a number that cannot be written in the form  $\frac{a}{b}$  where  $a$  and  $b$  are integers in their simplest form. Irrational numbers have infinitely many decimal places which do not repeat meaning they can not be expressed as a recurring decimal. Irrational numbers include certain well-known constants such as pi ( $\pi$ ) and the golden ratio ( $\phi$ ), as well as a group of numbers called surds. Surds are the square roots of numbers that are not perfect squares (eg. 2, 7, 11) and, since it is impossible to write all of the decimal places, they are written as  $\sqrt{2}$ ,  $\sqrt{7}$ ,  $\sqrt{11}$  etc.

## Some Music Theory

### Notes in Music

Let us start with a piano. Each of the keys are assigned letters—notes—with each key representing one note. The white keys follow the pattern CDEFGABC and so on (Figure 1).

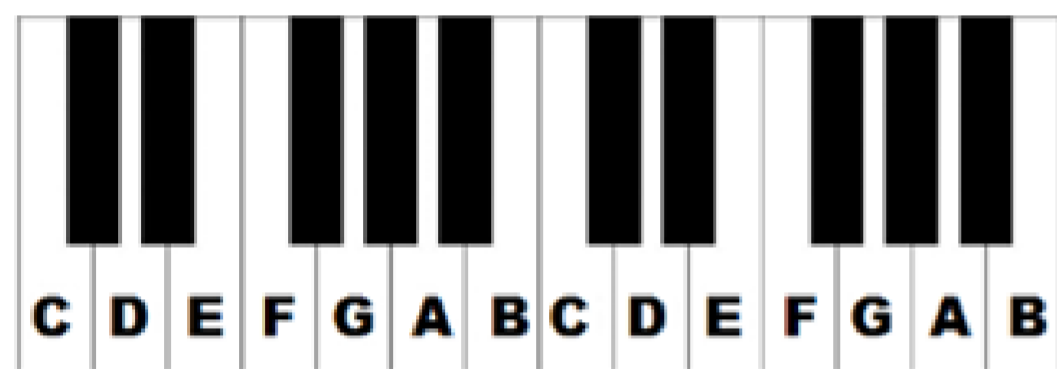


Figure 1: Notes on a Piano [8]

Each key moved to the right is called a semitone and moving up two keys is called a tone. The gap from one note to the next note with the same letter is called an octave and contains twelve semitones. The black keys are assigned letters with either sharp  $\sharp$  or flat  $\flat$ . The letter they are assigned depends on the white keys either side of it. Sharp is where the pitch of the note has been moved up a semitone, whereas flat is where it has been moved down a semitone. For example, as seen below (Figure 2) one key up from C is  $C\sharp$ . Likewise, one key down from B is  $B\flat$ .

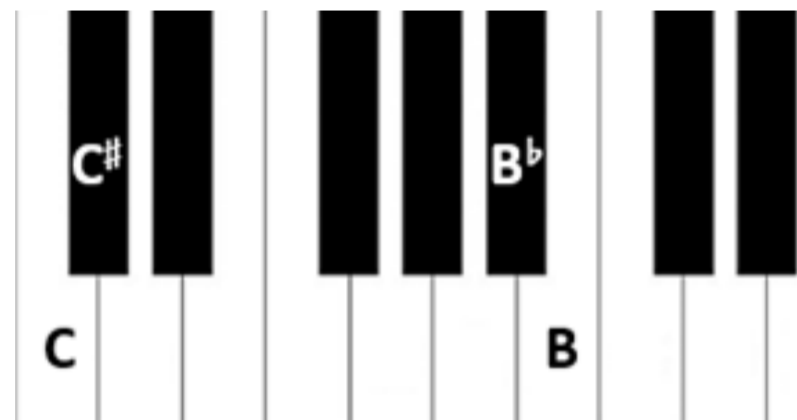


Figure 2: Sharps and Flats [7]

This system means that black keys each have two names so  $C\sharp$  can also be called  $D\flat$  because it can be reached by going up from C or down from D. Some sharps and flats aren't used.  $F\flat$  is the same as E so it's always known as E. Likewise with  $B\sharp$  which is the same as C,  $E\sharp$  which is the same as F and  $C\flat$  which is the same as B. Notes that are not sharp or flat are known as natural  $\natural$ .

### Scales

A scale is defined as "an arrangement of the notes in any system of music in ascending or descending order of pitch" and they are incredibly important to any piece of music [3]. The note on which a scale starts is called the *tonic*, and it is this that the scale is named after (eg. The D major scale starts on a D). Any scale can be named after any note.

The simplest of the scales is the major scale and the simplest of these is the C major scale.

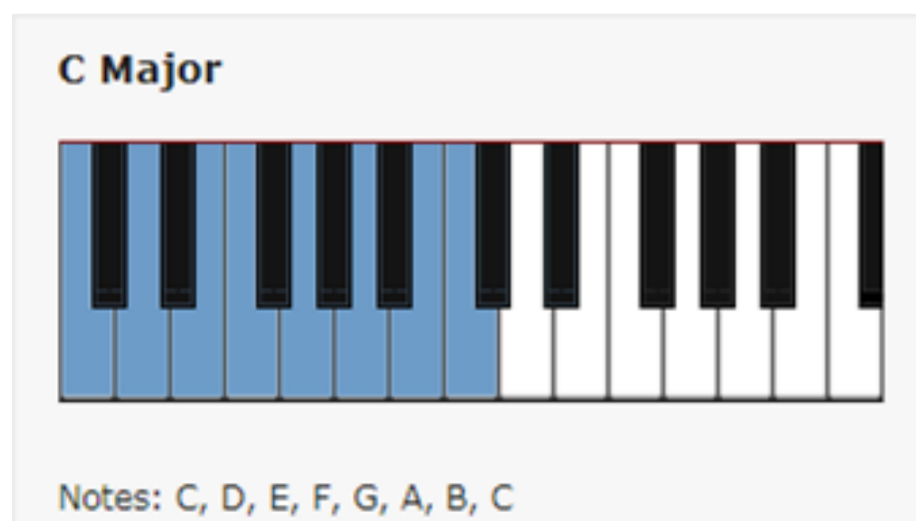


Figure 3: C Major Scale [6]

As shown in Figure 3, the C major scale contains the notes CDEFGABC and uses only the white keys of a piano. The intervals between notes in a major scale follow the pattern:

tone	tone	semitone	tone	tone	tone	semitone
2	2	1	2	2	2	1

This can also be modelled algebraically as:

$$n + 0, n + 2, n + 4, n + 5, n + 7, n + 9, n + 11, n + 12$$

where  $n$  is the tonic note and the number added to it represents the number of semitones counted up from the tonic.

There are many more types of scales including:

- Natural minor
- Melodic and harmonic minors
- Pentatonic (major and minor)
- Jazz
- Blues

## Initial Research

As part of my research, I used YouTube to find examples of where people have created music from pi and other irrational numbers in a similar manner to how I aim to do so. I also used these as a way to influence my decision about which scale to use. I analysed their process for assigning notes to the digits as well as looking at how many digits, what types of instruments and any other techniques the artist used.

In the first example the artist assigns the digits 0-9 to the C major Pentatonic Scale[9]. They also assigned pairs of digits to different durations of notes—for example, 0 and 1 meant the note lasted for two whole notes and 6 and 7 meant the note lasted for half a note. For the first 997 digits of pi (3.14159) the artist took the first digit to assign the note and the second digit for its duration. They played a simple chord progression on piano and the sequence of notes from pi over the top.

Other examples took a more simple approach of having all the notes created from numbers played for the same length of time and many layered different instruments over the top of each other. One artist created songs from pi, phi ( $\phi$ ) and tau ( $\tau$ ) [2][1][3].

## Methodology

At first I decided to use the D Minor Pentatonic Blues Scale as my chosen scale. However, for reasons highlighted in the Evaluation section, I changed this to the C Major scale to create my piece. I assigned the digits 0-9 to the notes of the scale as follows:

C	D	E	F	G	A	B	C	D	E
0	1	2	3	4	5	6	7	8	9

I initially took the first 24 digits of  $\sqrt{2}$  and converted them into notes. I put these into MuseScore—which is a software for creating sheet music—and chose to hear them played back on a piano[4]. I used the time signature  $\frac{4}{4}$  (meaning there are four beats per bar) and each note lasted for one beat. In addition to this, I added 8 more digits of  $\sqrt{2}$  (now a total of 32 digits) so that the pattern lasted for 8 bars, which was easier with timing. I used this phrase as my starting point (Figure 4).



Figure 4: Screenshot from MuseScore

Next, I took the diatonic chords of the C major scale. This means taking the first note in the scale (C) and playing it at the same time as the notes that occur 3rd and 5th in the scale, in this case these are E and G.



Figure 5: C Major chord [5]

Using this technique and the C major scale you can create the chords C, D, Em, F, G, Am and Bdim. These are traditionally labelled with Roman numerals as follows:

Roman Numeral	I	ii	iii	IV	V	vi	vii°
Chord in C major scale	C	Dm	Em	F	G	Am	Bdim

where the capitals are used to represent major chords, lower case for minors and the ° for diminished. The difference between major and minor sonically is that major sounds 'happy' and minor sounds 'sad'. By far the most popular progression of these chords is I-V-vi-IV, which is the progression I chose to use. In the case of the C major scale, this gives the chords C, G, Am and F respectively. I added these chords in another line underneath the initial phrase in the following pattern (Figure 6):



Figure 6: Screenshot from MuseScore

I added many layers with different instruments at different speeds. I introduced each new layer after one complete repeat of the original phrase (every 8 bars). First I added a classical guitar playing the melody twice as fast as the piano—as seen on the fourth line of the staff (Figure 7)—then a violin playing and holding the chords for a full bar (line five of the staff). I then added a cello playing the  $\sqrt{2}$  melody at half the speed of the piano, only two notes per bar (bottom line of the staff).



Figure 7: Screenshot from MuseScore

At this point the piece needed a change: a bridge of sorts. The bridge starts with all of the instruments removed except for the violins, and a flute is introduced that plays the  $\sqrt{2}$  melody at the same pitch as the original piano.



Figure 8: Screenshot from MuseScore

After only four bars of this (half of the  $\sqrt{2}$  phrase) I brought the cello back but this time playing long root notes of the chords played by the violin, as shown in Figure 9.



Figure 9: Screenshot from MuseScore

Next, I brought the piano riff back in to play in unison with the flute. Then I brought the guitar back, playing the riff at twice the speed of the piano, as before. At the same time, I changed the cello back to playing the  $\sqrt{2}$  phrase at half the speed of the piano. As my last change before the end of the piece I added the piano chords back in. All of the instruments play the phrase once through together.

I spent quite a while trying to perfect the ending of the piece. I knew that I wanted it to end on a long note but was unsure about which note and which instruments to play it. It made sense for the note to be a C, it being the next chord in the pattern. After some experimenting I settled on a rest for a full bar and then the violin, the cello and the flute playing a C for two bars (Figure 10).



Figure 10: Screenshot from MuseScore

## Evaluation

The scale I initially chose (D Minor Pentatonic Blues) produced a disjointed, tuneless melody that was an assault on the ears. The reason for this is that blues music is written to sound slightly sonically incoherent with '7th' chords and 'blue notes' used to create a unique sound. However, this incoherence is deliberate and carefully constructed so trying to create this sound from a set of random notes is almost impossible. Blues also incorporates an element of improvisation, which is lost in this formulaic way of creating music.

Conversely, the second scale I used (C major) worked extremely well: in fact, better than expected. The simplicity of this scale allowed the randomness to create something that sounded tuneful. Using this scale I successfully created a piece of music of an acceptable length to be considered finished. While it is not the most beautiful piece ever created, its sound does not betray the fact that its creation was due to a random string of digits.

## Further Work

Some ideas I have for further work include experimenting with different surds, combining multiple in one song, investigating how surds and their reciprocals sound when played together and branching out to more complex scales.

## References

- [1] Michael Blake. Song from phi. [https://www.youtube.com/watch?v=w\\_0b-X6DM14](https://www.youtube.com/watch?v=w_0b-X6DM14). Accessed 10 July 2018.
- [2] Michael Blake. Song from pi. <https://www.youtube.com/watch?v=wK7tq7LON8E>. Accessed 10 July 2018.
- [3] Michael Blake. Song from tau. <https://www.youtube.com/watch?v=3174T-3-59Q>. Accessed 10 July 2018.
- [4] MuseScore.org. MuseScore software. 2018. <https://musescore.org/hr/download/musescore.dmg>. Accessed 5 October 2018.
- [5] Dexter Music. Picture of a c chord. <https://www.dexter-music.jp/en/chord/c.html>. Accessed 14 December 2018.
- [6] Pianoscales. C major scale. <https://www.pianoscales.org/major.html>. Accessed 3 December 2018.
- [7] Yamaha. Picture of a blank piano, edited. <https://www.yamaha-keyboard-guide.com/piano-keyboard-diagram.html>. Accessed 10 September 2017.
- [8] Yamaha. Picture of a keyboard. <https://www.yamaha-keyboard-guide.com/piano-keys.html>. Accessed 7 September 2018.
- [9] YouTube. Song from pi. <https://www.youtube.com/watch?v=HV1-AjwD3wM>. Accessed 10 July 2018.

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