Methods for Synthesis of Almost Perfect and Complementary Phase Manipulated Signals with Lengths N≡2mod4

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Introduction

The so-called smart cars are in a rapid progress today. Theirs very important elements are the small radars, which in real time provide precise information about the spatial positioning of all objects on the road. Due to these reasons, in the paper two new methods for synthesis of periodic quasi binary almost perfect signals and complementary signals with lengths N≡2mod4 are developed. The usage of these signals by smart car radars could provide: very high object resolution, measurement accuracy of object spatial coordinates and electromagnetic compatibility with all other radio devices, working simultaneously.

Basic Requirements in the Synthesis of Phase Manipulated Signals

The digital signal processing in every communication receiver can be described by the following mathematical model:

$$\left[\sum_{i=0}^{N-1} s(i)x^{i}\right]\left[\sum_{i=0}^{N-1} v(i)x^{-i}\right] = \\ = \sum_{i=0}^{N-1} q_{sv}(i)x^{i} \, mod(x^{N}-1) \qquad (1)$$

In (1) the complex numbers $s(i), i = 0, 1, \ldots, N-1$ are the complex envelopes of the elementary phase symbols (or chips) with duration τ_{ch}, forming the PM signal $\{s(i)\}_{i=0}^{N-1}$, radiated by the transmitter.

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Analogously, the complex numbers $v(i), i = 0, 1, \dots, N-1$ and $q_{sv}(i), i = 0, 1, \dots, N-1$ are the samples of the finite response filter (FIR), used in the receiver, and the periodic cross-correlation function (PCCF) of digital signals $\{s(i)\}_{i=0}^{N-1}$ and $\{v(i)\}_{i=0}^{N-1}$ respectively.

As only the so-called Barker signal with length $N = 4$ has ideal periodic autocorrelation function (PACF), which side-lobes have zero magnitude, in the paper two new methods for synthesis of quasi binary almost perfect signals and complementary signals with lengths $N \equiv 2 mod 4$ are developed.

A Method for Synthesis of Quasi Binary Almost Perfect Signals with Lengths N≡2mod4

Sidelnikov and Lempel, Cohn, Eastman introduced independently a class of binary nearly perfect signals, which in sequel are named Sidelnikov-Lempel-Cohn-Eastman signals for brevity. The synthesis of these signals can be described as follows. Let the binary signal length be

$$N = p^m - 1 \equiv \begin{cases} 0 mod 4, \\ 2 mod 4, \end{cases} \tag{8}$$

where p is arbitrary odd prime and m is arbitrary positive integer. Then, the subset $D = \{d(l)\}_{l=0}^{N/2-1}$ of Z_N, defined by:

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$$d(l) = ind_\theta\left(\theta^{2l+1} - 1\right), l = 0, 1, \dots, N/2 - 1, \quad (9)$$

is an almost difference set (ADS) with parameters:

$$N = p^m - 1, k = \frac{N}{2},$$

$$\lambda = \begin{cases} \frac{N-4}{4}, N \equiv 0 mod 4, \\ \frac{N-2}{4}, N \equiv 2 mod 4, \end{cases} t = \begin{cases} \frac{N}{4}, N \equiv 0 mod 4, \\ \frac{3N-2}{4}, N \equiv 2 mod 4. \end{cases} \quad (10)$$

In (9) θ is an arbitrary primitive element of $GF(p^m)$ and $d(l) = ind_\theta\left(\theta^{2l+1} - 1\right)$ means that $\theta^{d(l)} = \theta^{2l+1} - 1$.

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From (10) and (7) it follows that in the case $N \equiv 2mod4$ the binary almost perfect signal, obtained from ADS (9) by the "classic" coding rule, has a two-level PACF $\{q_{ss}(i)\}_{i=0}^{N-1}$ with side-lobes:

$$Q_a = N - 4\left(\frac{N}{2} - \frac{N-2}{4}\right) = -2, t = \frac{3N-2}{4},$$
$$Q_b = N - 4\left(\frac{N}{2} - \frac{N-2}{4} - 1\right) = 2, t = \frac{N-2}{4}. \qquad (11)$$

As seen, this PACF weakly resembles a delta-pulse. Due to this fact in the paper it is substantiated that the "classic" code rule have to be transformed as follows:

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$$s'(i) = \begin{cases} +1, i \notin D; \\ x = e^{j\varphi} = cos\varphi + jsin\varphi, i \in D. \end{cases} \quad (12)$$

In (12) $j = \sqrt{-1}$ and the choice of the phase angle φ provides elimination of the side-lobes $Q_a = -2$. Namely:

$$\varphi = arccos\left[1 - \frac{N}{2(k-\lambda)}\right] = arccos\left(\frac{2-N}{N+2}\right). \quad (16)$$

The coding rule (12) generates quasi binary signals, which PACFs resembles a delta-pulse much more than the PACFs of the Sidelnikov-Lempel-Cohn-Eastman signals. This fact is illustrated on Fig. 1.

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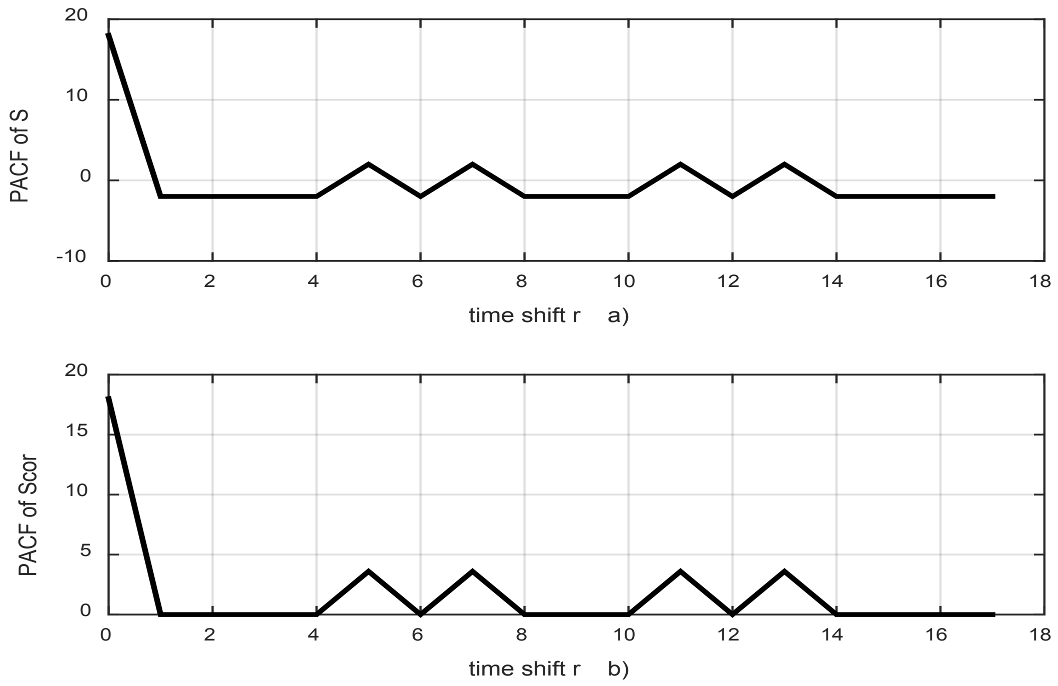

Fig. 1. The PACF (a) of the Sidelnikov-Lempel-Cohn-Eastman signal and the PACF (b) of the derivative quasi binary nearly perfect signal with parameters $p = 19, m = 1, \theta = 2, N = 18$

A Method for Synthesis of Quaternary Complementary Signals with Lengths N≡2mod4

The exhaustive computer modelling and survey, conducted by the authors, show that the positive side-lobes of PACF of the quasi binary signal $\{s'(i)\}_{i=0}^{N-1} = \{s_1(i)\}_{i=0}^{N-1}$, generated by the coding rule (12), are placed only on odd positions. This allows the quasi binary signal $\{s'(i)\}_{i=0}^{N-1} = \{s_1(i)\}_{i=0}^{N-1}$, generated by the coding rule (12), and the derivative quaternary PM signal

$$s_2(i) = (-1)^i s_1(i),\ i = 0, 1, \ldots, N-1. \qquad (20)$$

to be used as counterparts of a complementary pair of PM signals. This fact is illustrated on Fig. 2 .

A Method for Synthesis of Quaternary Complementary Signals with Lengths N≡2mod4

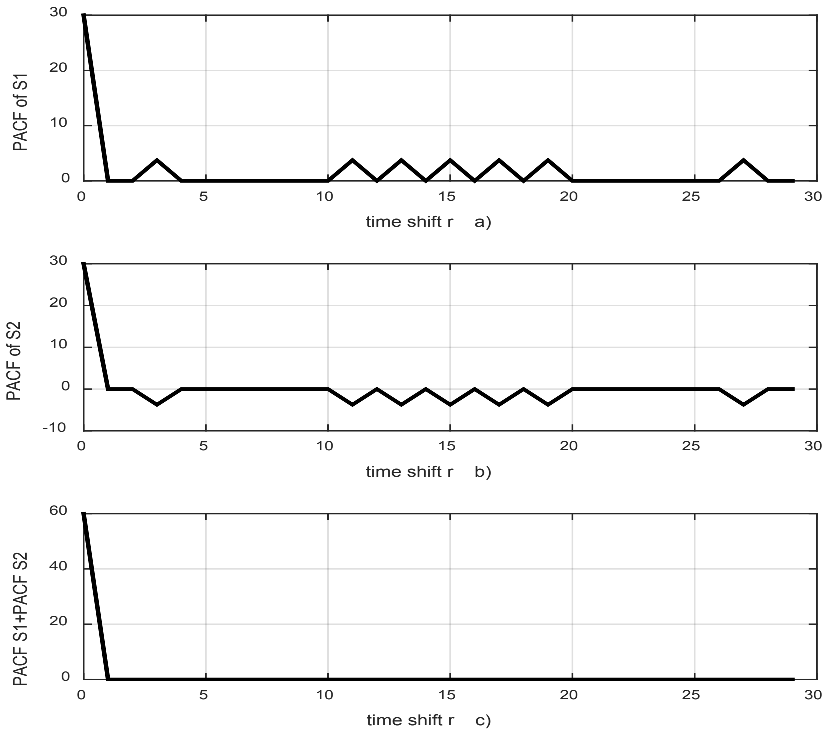

Fig. 2. The PACF (a) of the quasi binary nearly perfect signal $\{s_1(i)\}_{i=0}^{30}$ with parameters $p = 31, m = 1, \theta = 3, N = 30$, the PACF (b) of the derivative quaternary signal $\{s_2(i)\}_{i=0}^{30}$ and the sum (c) of PACFs of signals $\{s_1(i)\}_{i=0}^{30}$ and $\{s_2(i)\}_{i=0}^{30}$

Conclusion

In the paper two novel methods for synthesis of periodic quasi binary almost perfect signals and complementary signals with lengths $N \equiv 2 \bmod 4$ are developed.

Due to their positive features, signals, generated by methods, substantiated in the paper, could be used successfully in small radar sensors and in contemporary control systems, exploited in smart cars as well as in unmanned (including aerial) vehicles.