SEISMIC DESIGN GUIDE FOR MASONRY BUILDINGS

CHAPTER 4

Donald Anderson *Svetlana Brzev*

Canadian Concrete Masonry Producers Association

Canadian Concrete Masonry Producers Association
Region 6 of the National Concrete Masonry Association
CCMPA
Building Strength

April 2009

DISCLAIMER

While the authors have tried to be as accurate as possible, they cannot be held responsible for the designs of others that might be based on the material presented in this document. The material included in this document is intended for the use of design professionals who are competent to evaluate the significance and limitations of its contents and recommendations and able to accept responsibility for its application. The authors, and the Canadian Concrete Masonry Producers Association, disclaim any and all responsibility for the applications of the stated principles and for the accuracy of any of the material included in the document.

AUTHORS

Don Anderson, Ph.D., P.Eng.
Department of Civil Engineering,
University of British Columbia
Vancouver, BC

Svetlana Brzev, Ph.D., P.Eng.
Department of Civil Engineering
British Columbia Institute of Technology
Burnaby, BC

TECHNICAL EDITORS

Gary Sturgeon, P.Eng., Director of Technical Services, CCMPA
Bill McEwen, P.Eng., LEED AP, Executive Director, Masonry Institute of BC
Dr. Mark Hagel, EIT, Technical Services Engineer, CCMPA

GRAPHIC DESIGN

Natalia Leposavic, M.Arch.

COVER PAGE

Photo credit: Bill McEwen, P.Eng.
Graphic design: Marjorie Greene, AICP

COPYRIGHT

Canadian Concrete Masonry Producers Association
P.O. Box 54503, 1771 Avenue Road
Toronto, ON M5M 4N5
Tel: (416) 495-7497
Fax: (416) 495-8939
Email: information@ccmpa.ca
Web site: www.ccmpa.ca

The Canadian Concrete Masonry Producers Association (CCMPA) is a non-profit association whose mission is to support and advance the common interests of its members in the manufacture, marketing, research, and application of concrete masonry products and structures. It represents the interests of Region 6 of the National Concrete Masonry Association (NCMA).

Contents Summary

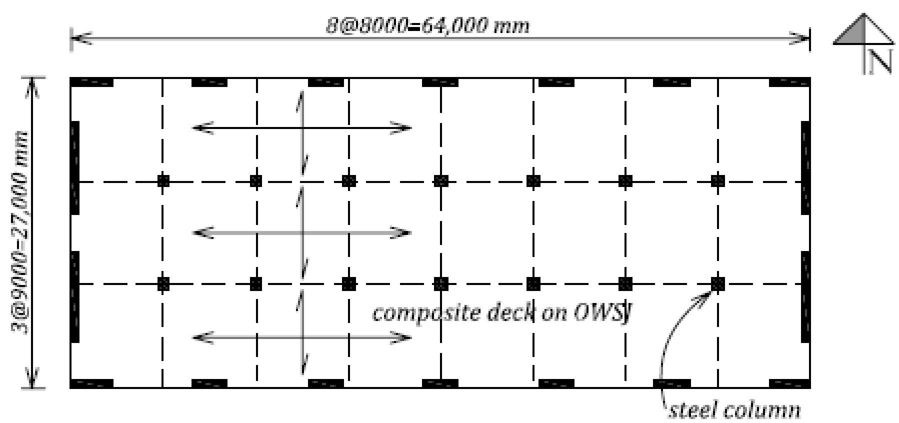
TABLE OF CONTENTS – CHAPTER 4

4 Design Examples

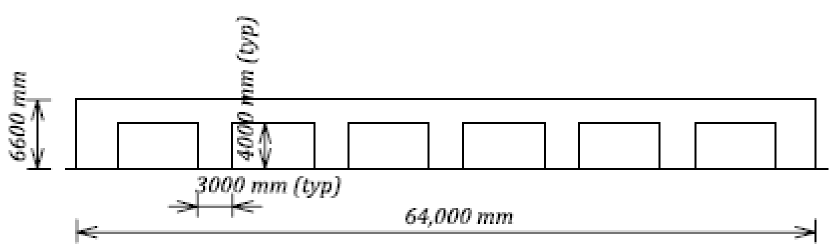
<u>EXAMPLE 1</u>: Seismic load calculation for a low-rise masonry building to NBCC 2005

Consider a single-storey warehouse building located in Mississauga, Ontario. The building plan dimensions are 64 m length by 27 m width, as shown on the figure below. The roof structure consists of steel beams, open web steel joists, and a composite steel and concrete deck with 70 mm concrete topping. The roof is supported by 190 mm reinforced block masonry walls at the perimeter and interior steel columns. The roof elevation is 6.6 m above the foundation. The soil at the building site is classed as a Site Class D per NBCC 2005.

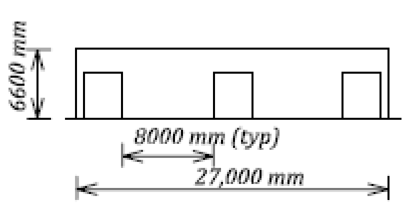
Calculate the seismic base shear force for this building to NBCC 2005 seismic requirements (considering the masonry walls to be detailed as "conventional construction"). Next, determine the seismic shear forces in the walls, including the effect of accidental torsional eccentricity. Assume that the roof acts like a rigid diaphragm.



Plan



North and South Elevations



East and West Elevations

SOLUTION:

1. Calculate the seismic weight W (NBCC 2005 Cl.4.1.8.2)

a) Roof loads:

- Snow load (Mississauga, ON) W_s = 0.25*(1.1*0.8+0.4)= 0.32 kPa

(25% of the total snow load is used for the seismic weight)

- Roof self-weight (including beams, trusses, steel deck, roofing, insulation, and 65 mm concrete topping) W_D = 2.60 kPa

Total roof seismic weight W_{roof} = (0.32kPa+2.60kPa)(64.0m*27.0m)= 5046 kN

b) Wall weight:

Assume solid grouted walls w = 4.0 kN/m^2

(this is a conservative assumption and could be changed later if it is determined that partially grouted walls would be adequate)

The usual assumption is that the weight of all the walls above wall midheight is part of the seismic weight (mass) that responds to the ground motion and contributes to the total base shear.

Tributary wall surface area:

- North face elevation = 0.5*7*3.0m*6.6m + (64m-7*3m)*(6.6m-4.0m)= 181.1 m^2
- South face elevation (same as north face elevation) = 181.1 m^2
- East face elevation = 0.5*2*8.0m*6.6m + (27m-2*8m)*(6.6m-4.0m) = 81.4 m^2
- West face elevation (same as east face elevation) = 81.4 m^2

Total tributary wall area $Area$ = 525.0 m^2

Total wall seismic weight $W_{wall} = w * Area$ = 4.0*525.0= 2100 kN

The total seismic weight is equal to the sum of roof weight and the wall weight, that is,

$$W = W_{roof} + W_{wall} = 5046+2100 = 7146 \text{ kN} \approx 7150 \text{ kN}$$

2. Determine the seismic hazard for the site (see Section 1.5.2).

- Location: Mississauga, ON

  $S_a(0.2)$ = 0.31 (NBCC 2005 Appendix C, page C-21)

  $S_a(0.5)$ = 0.15

  $S_a(1.0)$ = 0.055

  $S_a(2.0)$ = 0.017

- Foundation factors

  F_a = 1.28 for $S_a(0.2)$ =0.31 and Site Class D (by interpolation from Table 1-10 or NBCC 2005 Table 4.1.8.4.B, since F_a = 1.3 for $S_a(0.2) \leq$ 0.25 and F_a = 1.2 for $S_a(0.2) =$ 0.50

  F_v = 1.4 for $S_a(1.0)$ = 0.055 and Site Class D (from Table 1-11 or NBCC 2005 Table 4.1.8.4.C), since F_v = 1.4 for $S_a(1.0) \leq$ 0.1

- Site design spectrum $S(T)$ (see Section 1.5.2)

For T =0.2 sec: $S(0.2) = F_a S_a(0.2)$= 1.28*0.31=0.40 $\boxed{S(0.2)=0.40}$

For T =0.5 sec: use the smaller of;

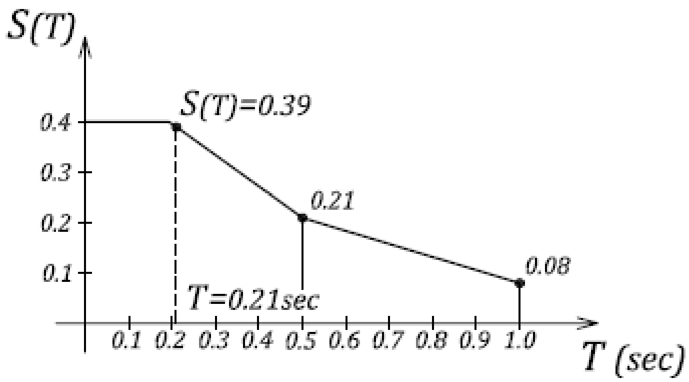
$$S(0.5) = F_v S_a(0.5) = 1.4*0.15 = 0.21 \qquad \text{or}$$

$S(0.5) = F_a S_a(0.2)$=1.28*0.31=0.40, thus $\boxed{S(0.5)=0.21}$

For T =1 sec: $S(1.0) = F_v S_a(1.0)$=1.4*0.055=0.08 $\boxed{S(1.0)=0.08}$

For T =2 sec: $S(2.0) = F_v S_a(2.0)$=1.4*0.017=0.024 $\boxed{S(2.0)=0.024}$

The site design spectrum $S(T)$ is shown below.



- Building period (T) calculation (see Section 1.5.4 and NBCC 2005 Cl.4.1.8.11.(3).c) for wall structures)

h_n = 6.6 m building height

$T = 0.05(h_n)^{3/4}$ = 0.21 sec

Then interpolate between $S(0.2)$ and $S(0.5)$ to determine the design spectral acceleration:
$S(T) = S(0.21)$= 0.39

3. Compute the seismic base shear (see Section 1.5.4)

The base shear is given by the expression (NBCC 2005 Cl.4.1.8.11)

$$V = \frac{S(T) M_v I_E}{R_d R_o} W$$

where

I_E = 1.0 (building importance factor, equal to 1.0 for normal importance, 1.3 for high importance, and 1.5 for post-disaster buildings)

M_v = 1.0 (higher mode factor, equal to 1.0 for $T \le 1.0$ sec, that is, most low-rise masonry buildings)

Building SFRS description: masonry structure – conventional construction (see Table 1-13 or NBCC 2005 Table 4.1.8.9), hence R_d = 1.5 and R_o = 1.5

The design base shear V is given by:

$$V = \frac{S(T) M_v I_E}{R_d R_o} W = \frac{0.39*1.0*1.0}{1.5*1.5} W = 0.17W$$

but not less than

$$V_{\min} = \frac{S(2.0) M_v I_E W}{R_d R_o} = \frac{0.024*1.0*1.0}{1.5*1.5} W = 0.0011W$$

and need not be taken more than

$V_{max} = \left(\frac{2S(0.2)}{3}\right)\left(\frac{I_E W}{R_d R_o}\right) = \left(\frac{2*0.40}{3}\right)\left(\frac{1.0}{1.5*1.5}\right)W = 0.12W$, provided $R_d \geq 1.5$.

The upper limit on the design seismic base shear governs and therefore
$V = 0.12W = 0.12*7150 = 858 \approx 860$ kN
Note that the upper limit on the base shear is often going to govern for low-rise masonry structures which have low fundamental periods. The lower bound value would generally only apply to very tall buildings.

4. Determine if the equivalent static procedure can be used (see Section 1.5.3 and NBCC 2005 Cl. 4.1.8.7).
According to the NBCC 2005, the dynamic method is the default method of determining member forces and deflections, but the equivalent static method can be used if the structure meets <u>any</u> of the following criteria:
(a) is located in a region of low seismic activity where the seismic hazard index $I_E F_a S_a(0.2) < 0.35$.
In this case, the seismic hazard index is $I_E F_a S_a(0.2)$=1.0*1.28*0.31=0.40 > 0.35
and so this criterion is not satisfied.
(b) is a regular structure less than 60 m in height with period T < 2 seconds in either direction.
This building is clearly less than 60 m in height and the period T < 2 sec (as discussed above). A structure is considered to be regular if it has none of the irregularities discussed in Table 1-15 of Section 1.5.10.1. A single storey structure by definition will not have any irregularities of Type 1 to 6. It does not have a Type 8 irregularity (non-orthogonal system) but could have a Type 7 irregularity (torsional sensitivity), and so this criterion may or may not be satisfied, depending on the torsional sensitivity.
(c) has any type of irregularity, other than Type 7, and is less than 20 m in height with period T < 0.5 seconds in either direction.
This structure satisfies the height and period criteria.

Since the criterion c) has been satisfied, the design can proceed by using the equivalent static analysis procedure. It will be shown later that, even when using a conservative assumption, the torsional sensitivity parameter B=1.2<1.7. Thus criterion b) would also be satisfied. For structures with the lateral resisting elements distributed around the perimeter walls the B value will almost always be less than 1.7.

5. Distribute the base shear force to the individual walls.
In this example, the structure is symmetric in each direction and so the centre of mass, C_M , and the centre of resistance, C_R , coincide at the geometric centre of the structure. One might argue that in this simple system with walls at only each side of the building, the system is statically determinate in each direction and the total shear on each side can be determined using statics. However, how much shear goes to each of the walls on a side depends on the relative stiffness of the walls, although once yielding occurs the force on each wall depends on the yield strength of the wall.

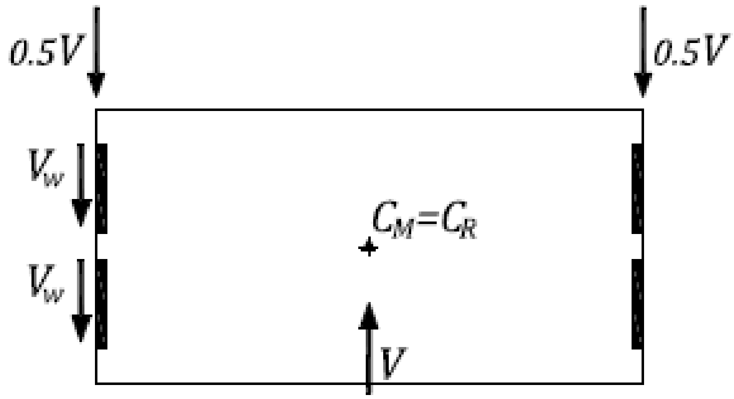
a) Seismic forces in the N-S direction - no torsional effects (seismic force is assumed to act through the centre of resistance)
Since it is assumed that the roof diaphragm is rigid, the forces are distributed to the walls in proportion to wall stiffness. All walls in the N-S direction have the same geometry (height, length, thickness) and mechanical properties and it can be concluded that these walls have the same stiffness.

As a result, equal shear force will be developed at each side. The force per side is equal to (see the figure):

$0.5V = 0.5 * 860 = 430$ kN

So, shear force in each of the two walls in the N-S direction is equal to:

$$V_V = \frac{0.5V}{2} = \frac{430}{2} = 215 \text{ kN}$$



b) Seismic forces in the N-S direction taking into account the effect of accidental torsion

The building is symmetrical in plan and so the centre of mass C_M coincides with the centre of resistance C_R (see Section 1.5.9 for more details on torsional effects). Therefore, there are no actual torsional effects in this building. However, NBCC 2005 Cl.4.1.8.11.(8) requires that torsional moments (torques) due to accidental eccentricities must be taken into account in the design. The forces due to accidental torsion can be determined by applying the seismic force at a point offset from the C_R by an accidental eccentricity $e_a = 0.1D_{nx}$, thereby causing the torsional moments equal to

$$T_x = \pm V(0.1D_{nx}) = \pm 860 * (0.1 * 64.0) = \pm 5504 \text{ kNm}$$

Note that $D_{nx} = 64.0$ m (equal to the total length of the structure in the East/West direction).

As a result of the accidental torsion, seismic shear forces resisted by each side of the building are different. These forces can be calculated by taking the sum of moments around the C_R (torsional moment created by force must be equal to the sum of moments created by the side forces). The resulting end forces are equal to $0.6V$ and $0.4V$, thereby indicating an increase in the end forces by $0.1V$ due to accidental torsion.

It should be noted that, in this example, accidental torsion would cause forces in the E-W walls as well because of the rigid diaphragm. But a conservative approach is to ignore the contribution of E-W walls and take all the torsional forces on the N-S walls.

The shear force in each N-S wall from accidental torsion is equal to:

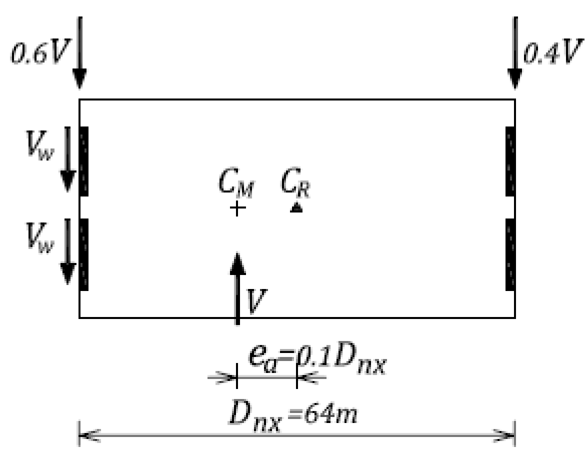
$$V_T = \frac{T/D_{nx}}{2} = \frac{5504/64}{2} = 43 \text{ kN}$$

Thus the maximum shear force in each of the two walls is the sum of the lateral component plus the torsional force,

$$V_W = V_V + V_T = 215 + 43 = 258 \text{ kN}$$

Note that the same result could be obtained by applying the lateral load through a point equal to the accidental eccentricity to one side of the centre of rigidity and then solving for the wall forces using statics (see the figure). This would show that

$$V_W = \frac{V}{2} * 0.6 = \frac{860}{2} * 0.6 = 258 \text{ kN}$$



Therefore, even though this building is symmetrical in plan, the accidental torsion causes increased seismic shear force in each wall of 43 kN, corresponding to a 20% increase compared to the design without torsion. However, this is based on the assumption that the N-S walls resist all the torsion. Walls in the E-W direction would also resist the torsional forces, and in this example the contribution to total torsional stiffness would be roughly the same for the E-W and N-S walls. Thus one could reduce the torsional forces on the N-S walls by roughly one half.

c) Seismic forces in the E-W walls

Seismic forces in the E-W walls can be determined in a similar manner. Since all walls in the E-W direction have the same geometry (height, length, thickness) and mechanical properties and consequently the same stiffness, the shear force will be equal at the East and West side. The force per side is equal to

$0.5V = 0.5 * 860 = 430$ kN

- Seismic forces in the E-W walls – torsional effects ignored

Shear force in each E-W wall is equal to (there are seven walls per side):

$$V_V = \frac{0.5V}{7} = \frac{430}{7} = 61 \text{ kN}$$

- Seismic forces in the E-W walls – torsional effects considered:

$$V_W = \frac{V}{7} * 0.6 = \frac{860}{7} * 0.6 = 74 \text{ kN}$$

6. Check whether the structure is torsionally sensitive (see Section 1.5.9.2).

NBCC 2005 Cl. 4.1.8.11.(9) requires that the torsional sensitivity B of the structure be determined by comparing the maximum horizontal displacement anywhere on a storey, to the average displacement of that storey. Torsional sensitivity is determined in a similar manner as the effect of accidental torsion, that is, by applying a set of a set of lateral forces at a distance of $\pm 0.1D_{nx}$ from the centre of mass C_M. In case of a rigid diaphragm, displacements are proportional to the forces developed in the walls. Therefore, B can be determined by comparing the forces at the sides of the building with/without the effect of accidental torsion.

The maximum displacement would be proportional to 0.6V, while the displacement on the other side would be proportional to 0.4V. Thus the average displacement is proportional to 0.5V. Thus

$$B = \frac{0.6V}{0.5V} = 1.2$$

Since B < 1.7, this building is not torsionally sensitive and the equivalent static analysis would have also been allowed under criterion b) as discussed in step 4 above.

7. Discussion

It was assumed at the beginning of this example that the roof structure can be modeled like a rigid diaphragm. If this roof was modeled like a flexible diaphragm, the shear forces in each N-S wall would be equal to 0.5V. From a reliability point of view, it does not seem quite right that the forces are smaller for a flexible diaphragm than a rigid one - it should be the other way around. On the other hand, the flexible diaphragm may have a longer period and the forces would be smaller (see Example 3 for a detailed discussion on rigid and flexible diaphragm models).

<u>EXAMPLE 2</u>: Seismic load calculation for a medium-rise masonry building to NBCC 2005

A typical floor plan and vertical elevation are shown below for a four-storey mixed use (commercial/residential) building located near the intersection of Granville Street and 41st Avenue in Vancouver, BC. The ground floor is commercial with a reinforced concrete slab separating it from the residential floors, which have lighter floor system consisting of steel joists supporting a composite steel and concrete deck. The front of the building is mostly glazing, which has no structural application.

First, determine the seismic force for this building according to the NBCC 2005 equivalent static force procedure, and a vertical force distribution in the E-W direction. Find the base shear and overturning moment in the E-W walls. Assume that the floors act as rigid diaphragms and that the strong N-S walls can resist the torsion.

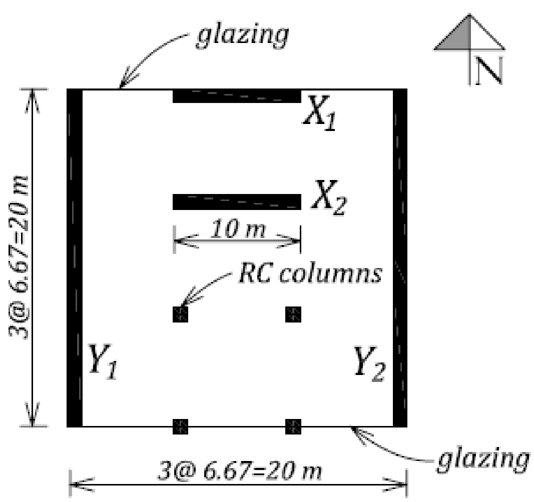
Next, consider the torsional effects in all walls and find the forces in the E-W walls. Compare the seismic forces obtained with and without torsional effects.

For the purpose of weight calculations, use 200 mm blocks for N-S walls and 300 mm blocks for E-W walls. All walls are solid grouted (this is a conservative assumption appropriate for a preliminary design) and the compressive strength f'_m is 10.0 MPa. Grade 400 steel has been used for the reinforcement. The building is of normal importance and is supported on Class C soil. Consider "limited ductility" reinforced masonry shear walls.

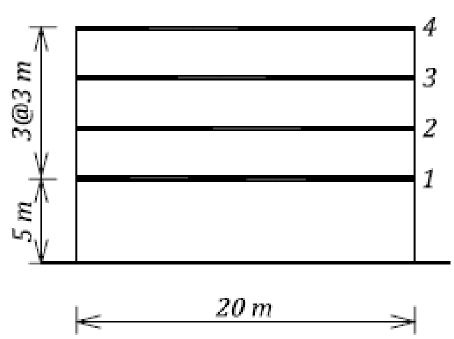
Movement joints are not to be considered in this example. Note that movement joints in the N-S walls would have caused slight changes in the stiffness values of these walls.

Specified loads (note that roof and floor loads include a 1 kPa allowance for partition walls and glazing):
4th floor (roof level) = 3 kPa
2nd and 3rd floor = 4 kPa
1st floor (concrete floor) = 6 kPa
25% snow load = 0.4 kPa

Note: 1 kPa = 1 kN/m^2



Plan



Elevation

<u>SOLUTION:</u>

1. Design assumptions

- Rigid diaphragm
- All walls are solid grouted

2. Calculate the seismic weight W (see Table 1-12 and NBCC 2005 Cl.4.1.8.2)

Wall weight:

N-S walls - 200 mm thick w = 4.18 kPa

E-W walls – 300 mm thick w = 6.38 kPa

Note that, for the purpose of seismic weight calculations, the length of a N-S wall is 20 m, while the length of an E-W wall is 10.0 m.

Seismic weight W_1:

$$W_1 = \left(\frac{5.0m}{2} + \frac{3.0m}{2}\right)(4.18kPa * 2 * 20m + 6.38kPa * 2 * 10.0m) + (6.0kPa)(20m * 20m) = 3579kN$$

Seismic weight W_2:

$$W_2 = \left(\frac{3.0m}{2} + \frac{3.0m}{2}\right)(4.18kPa * 2 * 20m + 6.38kPa * 2 * 10.0m) + (4.0kPa)(20m * 20m) = 2484kN$$

Seismic weight W_3 (same as W_2) :

$$W_3 = 2484kN$$

Seismic weight W_4:

$$W_4 = \left(\frac{3.0m}{2}\right)(4.18kPa * 2 * 20m + 6.38kPa * 2 * 10.0m) + (3.0kPa + 0.4kPa)(20m * 20m) = 1802kN$$

Note that the seismic weight for each floor level is the sum of the wall weights and the floor weight. 25% snow load was included in the roof weight calculation. One-half of the wall height (below and above a certain floor level) was considered in the wall area calculations.

The total seismic weight is equal to

$$W = W_1 + W_2 + W_3 + W_4 = 3579 + 2484 + 2484 + 1802 \cong 10350kN$$

3. Calculate the seismic base shear force (see Section 1.5.4).

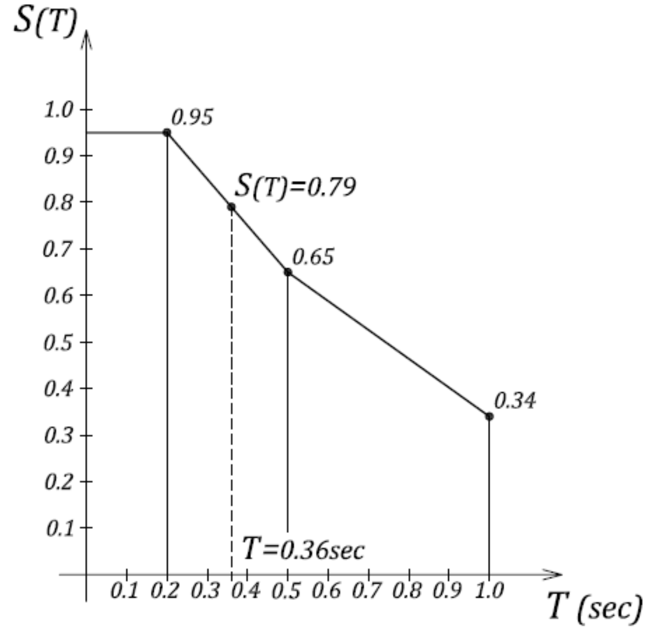
a) Find seismic design parameters used to determine seismic base shear.

- Location: Vancouver, BC (Granville and 41st Avenue)
  - $S_a(0.2)$ = 0.95 (NBCC 2005 Appendix C, page C-13)
  - $S_a(0.5)$ = 0.65
  - $S_a(1.0)$ = 0.34
  - $S_a(2.0)$ = 0.17
- Foundation factors
  - Site Class C: F_a = 1.0 (Table 1-10 or NBCC 2005 Table 4.1.8.4.B) and F_v = 1.0 (Table 1-11 or NBCC 2005 Table 4.1.8.4.C)
- I_E = 1.0 (normal importance building)
- M_v = 1.0 (higher mode factor, equal to 1.0 for $T \leq 1.0$ sec)
- Building SFRS description: masonry structure – limited ductility shear walls for building height of 14 m (see Table 1-13 or NBCC 2005 Table 4.1.8.9), hence

R_d = 1.5 and R_o = 1.5

- Building period (T) calculation (NBCC 2005 Cl.4.1.8.11.3.c) – wall structures
  h_n = 14.0 m building height
  $T = 0.05(h_n)^{3/4}$ = 0.36 sec
- Site design spectrum $S(T)$ (see Section 1.5.2)

Since the soil is characterized as Site Class C,
$S(T) = S_a(T)$



- For T =0.2 sec:

$S(0.2) = F_a S_a(0.2)$= 1.0*0.95=0.95 $\boxed{S(0.2)=0.95}$

- For T =0.5 sec:

$S(0.5) = F_v S_a(0.5)$=1.0*0.65=0.65 $\boxed{S(0.5)=0.65}$
or (smaller value governs)
$S(0.5) = F_a S_a(0.2)$=1.0*0.95=0.95

- For T =1 sec:

$S(1.0) = F_v S_a(1.0)$=1.0*0.34=0.34 $\boxed{S(1.0)=0.34}$

- For T =2 sec:

$S(2.0) = F_v S_a(2.0)$= 1.0*0.17=0.17 $\boxed{S(2.0)=0.17}$

Building period T = 0.36 sec, so interpolate between $S(0.2)$ and $S(0.5)$, hence $\boxed{S(T)= 0.79}$

b) Compute the design base shear (NBCC 2005 Cl.4.1.8.11).

The design base shear V is determined according to the following equation:

$$V = \frac{S(T)M_v I_E}{R_d R_o} W = \frac{0.79*1.0*1.0}{1.5*1.5} W = 0.35W$$

Check the lower and upper bounds for the V value.

- Lower bound ($V_{\min}$) value (must be exceeded)

$$V_{\min} = \frac{S(2.0)M_v I_E W}{R_d R_o} = \frac{0.17*1.0*1.0}{1.5*1.5} W = 0.075W$$

- Upper bound ($V_{\max}$) value (base shear need not exceed this value)

$$V_{\max} = \left(\frac{2S(0.2)}{3}\right)\left(\frac{I_E W}{R_d R_o}\right) = \left(\frac{2*0.95}{3}\right)\left(\frac{1.0}{1.5*1.5}\right) W = 0.28W < 0.35W \quad \text{<= Governs}$$

Note that the upper bound base shear value can be used only when $R_d \geq 1.5$.

Therefore, the design seismic base shear is equal to
$V = 0.28W = 0.28*10350 = 2898 \approx 2900$ kN

4. Determine if the equivalent static procedure can be used (see Section 1.5.7 and NBCC 2005 Cl. 4.1.8.7).

According to the NBCC 2005, the dynamic method is the default method, but the equivalent static method can be used if the structure meets any of the following criteria:

(a) is located in a region of low seismic activity where $I_E F_a S_a(0.2) < 0.35$***,***

In this case, seismic hazard index $I_E F_a S_a(0.2)$=1.0*1.0*0.95=0.95 > 0.35 and so this criterion is not satisfied.

(b) is a regular structure less than 60 m in height with period T ***< 2 seconds in either direction,***

This building is clearly less than 60 m in height and the period T < 2 sec (as discussed above). To confirm that this structure is regular, the designer needs to review the irregularities discussed in Section 1.5.10.1. It can be concluded that this building does not have any of the irregularity types identified by NBCC 2005 and so this criterion is satisfied.

(c) has any type of irregularity (other than Type 7 that requires the dynamic method if B >1.7), but is less than 20 m in height with period T ***< 0.5 seconds in either direction***

This is an irregular structure, but it is less than 20 m in height and the period is less than 0.5 sec. The torsional sensitivity B should be checked to confirm that B < 1.7 (see Section 1.5.9.2).

Since the criterion b) has been satisfied, the design can proceed by using the equivalent static analysis procedure.

5. Seismic force distribution over the building height (see Section 1.5.7).

According to NBCC 2005 Cl. 4.1.8.11.(6), the total lateral seismic force, V, is to be distributed over the building height in accordance with the following formula (see Figure 1-15):

$$F_x = (V - F_t) \cdot \frac{W_x h_x}{\sum_{i=1}^{n} W_i h_i}$$

where

F_x – seismic force acting at level x

F_t – a portion of the base shear to be applied in addition to force F_n at the top of the building.

In this case, F_t = 0 since the fundamental period is less than 0.7 sec.

Interstorey shear force at level x can be calculated as follows:

$$V_x = F_t + \sum_{x}^{n} F_i$$

Bending moment at level x can be calculated as follows:

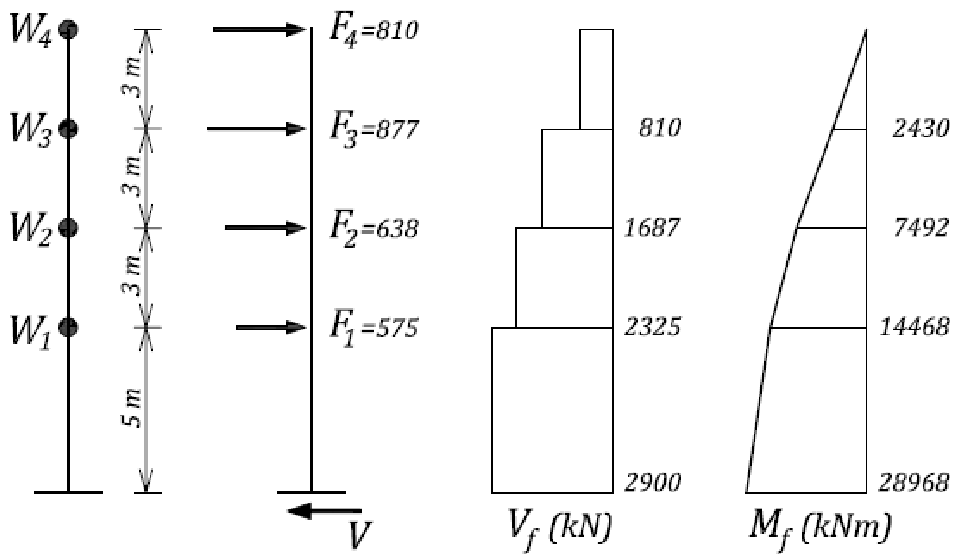
$$M_x = \sum_{i=x}^{n} F_i (h_i - h_x)$$

These calculations are presented in Table 1.

Table 1. Distribution of Seismic Forces over the Wall Height

Level	h_x (m)	W_x (kN)	$W_x h_x$	F_x (kN)	V_x (kN)	M_x (kNm)
4	14.0	1802	25228	810	810	0
3	11.0	2484	27324	877	1687	2430
2	8.0	2484	19872	638	2325	7492
1	5.0	3579	17895	575	2900	14468
$\sum$		10349	90319	2900		28968

Distribution of seismic forces over the building height and the corresponding shear and moment diagrams are shown on the figure below.



It is important to confirm that the sum of seismic forces F_x over the building height is equal to the base shear

$V_b = V =$ 2900 kN

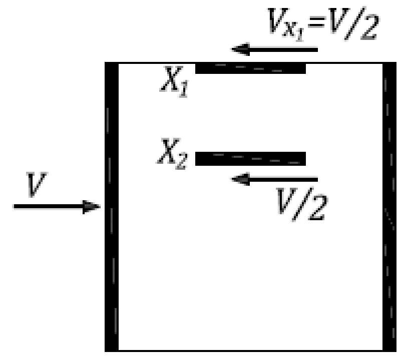
The bending moment at the base of the building, also called the base bending moment, is equal to

M_b = 28968 ≈ 29000 kNm.

6. Find the seismic forces in the E-W walls – torsional effects ignored.

Due to asymmetric layout of the E-W walls, the centre of mass C_M in the building under consideration does not coincide with the centre of resistance C_R, hence there are torsional effects in all walls. However, since the N-S walls are significantly more rigid compared to the E-W walls, it can be assumed that the N-S walls will resist the torsional effects (see step 8 for a detailed discussion). As a consequence, it can be assumed that the base shear force in the E-W direction is equally divided between the two E-W walls (see the figure on the next page), that is,

$$V_{xo} = \frac{V}{2} = \frac{2900}{2} = 1450 \text{ kN}$$



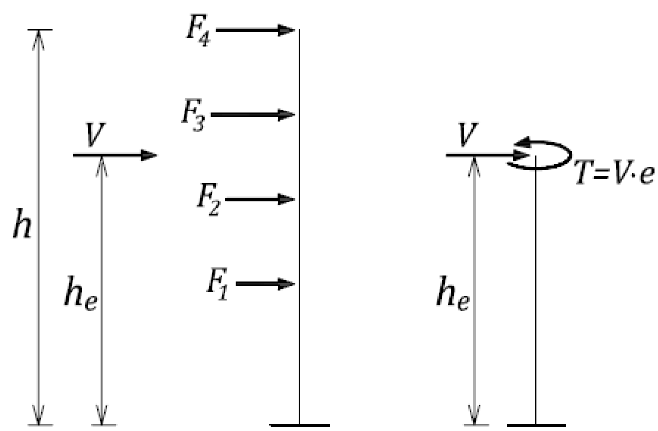
Similarly, the base bending moment in each wall is equal to

$$M_{bx} = \frac{M_b}{2} = \frac{29000}{2} = 14500 \text{ kNm}$$

7. Find the seismic forces in the E-W walls – torsional effects considered (see Section 1.5.9).

To determine the wall forces from the torsional forces a 3-D analysis should be made. Even though the walls are considered uniform over the entire height, the contribution of shear deformation relative to bending deformation is different over the height. An approximate method that does not require a 3-D analysis is to consider the structure as an equivalent single-storey structure. The entire shear is applied at the effective height, h_e, defined as the height at which the shear force V_f must be applied to produce the base moment M_f, that is,

$$h_e = \frac{M_f}{V_f} = \frac{29000}{2900} = 10.0 \text{ m}$$



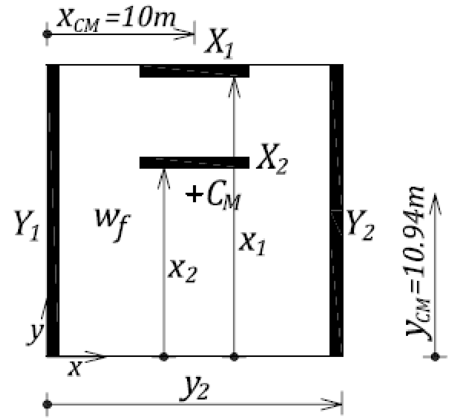
This model, although not strictly correct, will be used to determine the elastic distribution of the torsional forces as well as the displacements. The top displacement of the wall is assumed to be 1.5 times the displacement at the h_e height (see step 8 for displacement calculations).

Torsional moment (torque) is a product of the seismic force and the eccentricity between the centre of resistance (C_R) and the centre of mass (C_M), which will be calculated in the following tables.

First, the centre of mass will be determined, as shown on the figure on the next page. The calculations are summarized in Table 2.

Table 2. Calculation of the Centre of Mass (C_M)

Wall	w_i (kN)	x_i (m)	y_i (m)	$w_i * x_i$	$w_i * y_i$
X_1	733.7	10.00	20.00	7337	14674
X_2	733.7	10.00	13.33	7337	9780
Y_1	961.4	0	10.00	0	9614
Y_2	961.4	20.00	10.00	19228	9614
Floors	6960	10.00	10.00	69600	69600
$\sum$	10350			103502	113282



The C_M coordinates can be determined as follows:

$$x_{CM} = \frac{\sum_i w_i * x_i}{\sum_i w_i} = \frac{103502}{10350} = 10.00 \text{ m} \qquad y_{CM} = \frac{\sum_i w_i * y_i}{\sum_i w_i} = \frac{113282}{10350} = 10.94 \text{ m}$$

Next, the centre of resistance (C_R) will be determined, and the calculations are presented in Table 3, although because there are only two equal walls in each direction the C_R will lie between the walls.

Table 3. Calculation of the Centre of Resistance (C_R)

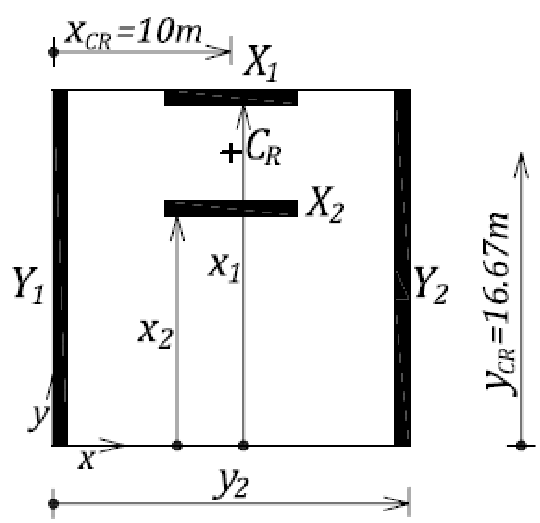
Wall	t (m)	h/l_w *	$K/(E_m \cdot t)$ **	K_x x10[3] (kN/m)	K_y x10[3] (kN/m)	x_i (m)	y_i (m)	$K_y \cdot x_i$ *10[3]	$K_x \cdot y_i$ *10[3]
X_1	0.29	1.0	0.143	352.5			20.00		7050.0
X_2	0.29	1.0	0.143	352.5			13.33		4699.0
Y_1	0.19	0.5	0.5		807.5	0		0	
Y_2	0.19	0.5	0.5		807.5	20.00		16150.0	
$\sum$				705.0	1615.0			16150.0	11750.0

Notes:

* - $h = h_e$ = 10.0 m effective wall height

** - see Table D-3

Note that the elastic uncracked wall stiffnesses K for individual walls have been determined from Table D-3, by entering appropriate height-to-length ratios. In this design, all walls and piers have been modelled as cantilevers (fixed at the base and free at the top) – see Section C.3 for more details regarding wall stiffness



calculations. The modulus of elasticity for masonry is $E_m = 8.5*10^6$ kPa (corresponding to f'_m of 10 MPa).

The C_R coordinates can be determined as follows (see the figure):

$$x_{CR} = \frac{\sum_i K_{yi} * x_i}{\sum_i K_{yi}} = \frac{16150*10^3}{1615*10^3} = 10 \text{ m}$$

$$y_{CR} = \frac{\sum_i K_{xi} * y_i}{\sum_i K_{xi}} = \frac{11750*10^3}{705*10^3} = 16.67 \text{ m}$$

Next, the eccentricity needs to be determined. Since we are looking for the forces in the E-W walls, we need to determine the actual eccentricity in the y direction (e_y), that is,

$$e_y = y_{CR} - y_{CM} = 16.67 - 10.94 = 5.73 \text{ m}$$

In addition, the accidental eccentricity needs to be considered, that is,

$$e_a = \pm 0.1 D_{ny} = \pm 0.1 * 20 = \pm 2.0 \text{ m}$$

The total maximum eccentricity in the y-direction is equal to

$$e_{ty1} = e_y + e_a = 5.73 + 2.0 = 7.73 \text{ m}$$

or

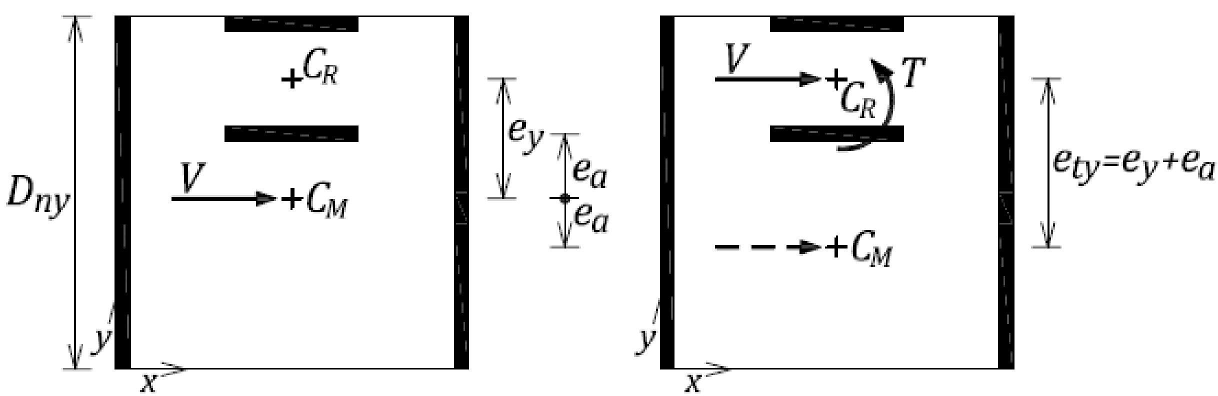
$$e_{ty2} = e_y - e_a = 5.73 - 2.0 = 3.73 \text{ m}$$

Note that the latter value does not govern and will not be considered in further calculations.

Torsional moment is determined as a product of the shear force and the eccentricity, that is,

$$T = V * e_{ty1} = 2900 * 7.73 = 22417 \text{ kNm}$$

Torsional effects are illustrated on the figure below.



Seismic force in each wall has two components: translational (no torsional effects) and torsional, that is,

$$V_i = V_{io} + V_{it}$$

where

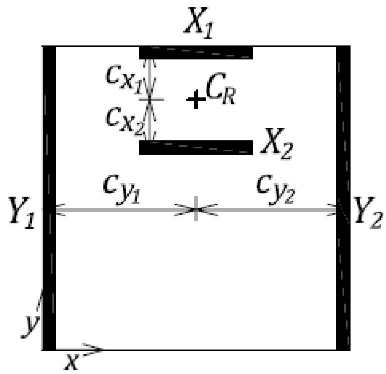
$$V_{io} = V * \frac{K_i}{\sum K_i} \quad \text{translational component}$$

and

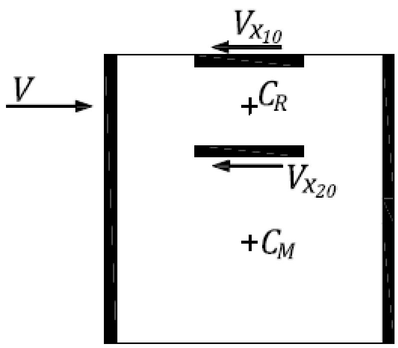
$V_{it} = \frac{T * c_i}{J} * K_i$ torsional component

$J = \sum K_{xi} \cdot c_{xi}^{\;2} + \sum K_{yi} \cdot c_{yi}^{\;2} = 169 * 10^6$ torsional stiffness (see Table 4)

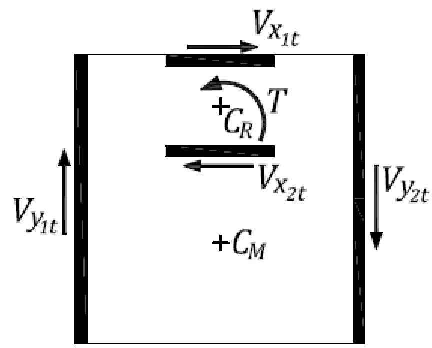
c_{xi}, c_{yi} - distance of the wall centroid from the centre of resistance (C_R) (see the figure below)



Translational and torsional force components for the individual walls are shown below.



Translational Force Components



Torsional Force Components

Calculation of translational and torsional forces is presented in Table 4.

Table 4. Seismic Shear Forces in the Walls due to Seismic Load in the E-W Direction

Wall	K_x *10³ (kN/m)	K_y *10³ (kN/m)	c_i (m)	$\sum K_i \cdot c_i^{\;2}$ *10⁶	$\frac{K_x}{\sum K_x}$	V_{xo} (kN)	V_{xt} (kN)	V_{total} (kN)
X_1	352.5		-3.33	3.84	0.5	1450	-154	1296
X_2	352.5		3.33	3.84	0.5	1450	154	1604
Y_1		807.5	-10.00	80.80			-1070	-1070
Y_2		807.5	10.00	80.80			1070	1070
$\sum$	705.0	1615.0		169.0				

It can be concluded from the above table that the maximum force in the E-W direction is equal to 1604 kN. This is an increase of only 11% as compared to the total force of 1450 kN obtained ignoring torsional effects.

It can be noted that the contribution of E-W walls to the overall torsional moment T of 22417 kNm is not significant (see Table 4).

$T_{E-W} = 154kN * 3.3m + 154kN * 3.3m \cong 1017kNm$

because

$T_{E-W} / T = 1017 / 22417 = 0.045 \approx 5\%$

this shows that the E-W walls contribute only 5% to the overall torsional moment.

The contribution of N-S walls to the overall torsional moment is as follows:

$T_{N-S} = 1070kN * 10m + 1070kN * 10m = 21400kNm$

and

$T_{N-S} / T = 21400 / 22417 \approx 95\%$

and

$T = T_{E-W} + T_{N-S} = 1017 + 21400 \approx 22417kNm$ (this is also a check for the torsional forces)

Therefore, the assumption that the torsional effects are resisted by N-S walls only is reasonable, since these walls contribute approximately 95% to the overall torsional resistance.

8. Calculate the displacements at the roof level (consider torsional effects).

Approximate deflections in the E-W walls can be determined according to the procedure outlined below. It should be noted that the force distribution calculations have been performed using elastic wall stiffnesses obtained from Table D-3. It is expected that the walls are going to crack during earthquake ground shaking; this will cause a drop in the wall stiffnesses. For the purpose of deflection calculations, we are going to use a reduction in the elastic stiffness (K) value to account for the effect of cracking.

a) The reduced stiffness to account for the effect of cracking (see Section C.3.5)

The reduced stiffness for walls X_1 *and* X_2 will be determined according to equation (15) from Section C.3.5, that is,

$$K_{ce} = (\frac{100}{f_y} + \frac{P_f}{f'_m A_e}) K_c$$

where

K_c is elastic uncracked stiffness

$P_f = (2 * 6.67 * 6.67)(3.0 + 2 * 4.0 + 6.0) = 1513$ kN (axial force due to dead load in wall X_2)

$A_e = (290 * 10^3) * 10.0 = 290 * 10^4$ mm^2 (effective cross sectional area for 290 mm block wall, solid grouted, length 10.0 m; see Table D-1 for A_e values for the unit wall length)

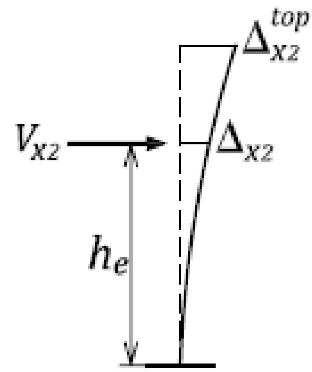
f'_m =10.0 MPa

$f_y = 400$ MPa (Grade 400 steel)

thus

$$K_{ce} = (\frac{100}{400} + \frac{1513 * 10^3}{10.0 * 290 * 10^4}) K_c = 0.3 K_c$$

b) The translational displacement in the walls X_1 and X_2 can be calculated as follows

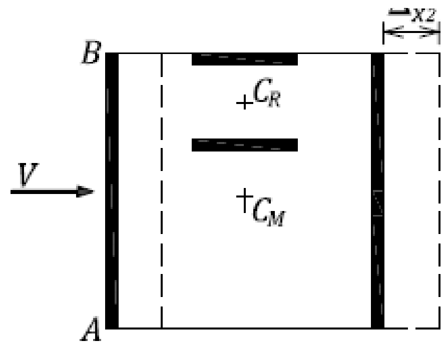


$$\Delta_{X20} = \frac{V_{X2o}}{0.3K_{X2}} = \frac{1450kN}{0.3*352.5*10^3 kN/m} = 13.7mm$$

According to NBCC 2005 Cl. 4.1.8.13, these deflections need to be multiplied by the $R_d R_o / I_E$ ratio (see Section 1.5.11). In this case, I_E = 1.0, and so

$$\Delta_{X20} = (13.7mm)R_d R_o = 13.7*1.5*1.5 = 30.8mm$$

Since the previous analysis assumed that the seismic force acts at the effective height h_e, the displacement at the top of the wall will be larger (see the figure). The top displacement can be calculated by deriving the displacement value at the tip of the cantilever; alternatively, an approximate factor of 1.5 can be used as follows:

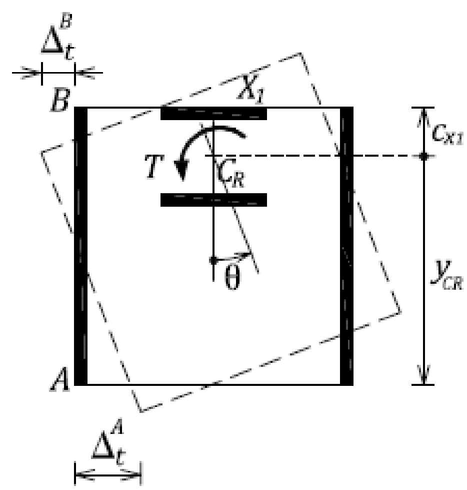


$$\Delta_{X20}^{top} = 1.5*\Delta_{x2} = 1.5*30.8mm \approx 46mm$$

Since this is a rigid diaphragm, it can be assumed that the translational displacements are equal at a certain floor level – let us use point A at the South-East corner as a reference (see the figure).

c) The torsional displacements can be calculated as follows:

Torsional rotation of the building θ can be determined as follows, considering the reduced torsional stiffness to account for cracking (same as discussed in step a) above):



$$\theta = \frac{T}{J} = \frac{22417kNm}{0.3*169*10^6} = 4.421*10^{-4}\text{ rad}$$

where (see the step 7 calculations)

$T = 22417$ kNm torsional moment

$J = 169*10^6$ elastic torsional stiffness

The maximum torsional displacement at the South-East corner in the X direction (see point A on the figure):

$$\Delta^A{}_t = \theta * Y_{CR} = 4.421*10^{-4}*16.67m = 7.4mm$$

Similarly as above, these displacements need to be multiplied by $R_d R_o / I_E$ and also by 1.5 to determine the displacement at the top of the roof, and so

$$\Delta^A{}_t{}^{top} = 1.5*7.4*R_d R_o \approx 25.0mm$$

d) Finally, the total maximum displacement at the roof level (at point A) is equal to:

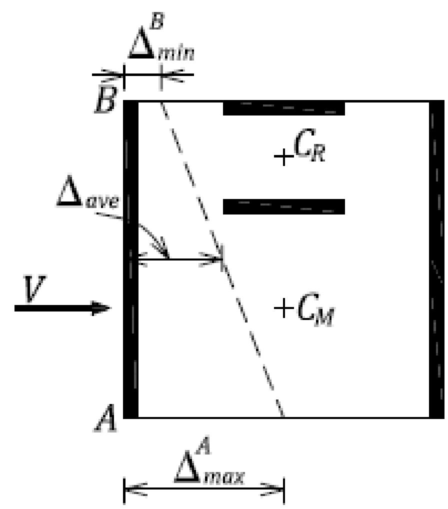
$$\Delta^A{}_{\max} = \Delta_{X2}{}^{top} + \Delta^A{}_t{}^{top} = 46 + 25 = 71mm$$

9. Check whether the building is torsionally sensitive.

NBCC 2005 Cl. 4.1.8.11.(9) requires that the torsional sensitivity B of the structure be determined by comparing the maximum horizontal displacement anywhere on a storey to the average displacement of that storey (see Section 1.5.9.2 and Figure 1-19). This should be done for every storey, but in this case will only be done for the one storey as the remaining storeys will have similar B values because of the vertical uniformity of the walls. Torsional sensitivity is determined in a similar manner like the effect of accidental torsion, that is, by applying a set of lateral forces at a distance of $\pm 0.1D_{nx}$ from the centre of mass C_M. Since the purpose of this evaluation is to compare deflections at certain locations relative to one another, it is not critical to use cracked wall stiffnesses.

In this case, the total maximum displacement at point A was determined in step 8 above, that is,
$\Delta^A{}_{\max} = 71mm$
We need to determine the displacement at other corner (point B), that is, the minimum displacement. This can be done as follows:
Translational component:
$\Delta^B{}_0 = \Delta_{X20}{}^{top} = 46mm$
Torsional component:
$\Delta_t = \theta * c_{X1} = 4.421*10^{-4} * 3.3m \approx 1.5mm$
These displacements need to be multiplied by $R_d R_o / I_E$ and also by 1.5 to determine the displacement at the top of the roof, and so
$\Delta^B{}_t = 1.5*1.5*R_d R_o = 5mm$
Since the direction of torsional displacements is opposite from the translational displacements, it follows that
$\Delta^B{}_{\min} = \Delta^B{}_o - \Delta^B{}_t = 46 - 5 = 41mm$
The average displacement at the roof level in the E-W direction (see the figure showing the displacement components):

$$\Delta_{ave} = \frac{\Delta^A{}_{\max} + \Delta^B{}_{\min}}{2} = \frac{71+41}{2} = 56mm$$



$$B = \frac{\Delta_{\max}}{\Delta_{ave}} = \frac{71.0}{56.0} = 1.27$$

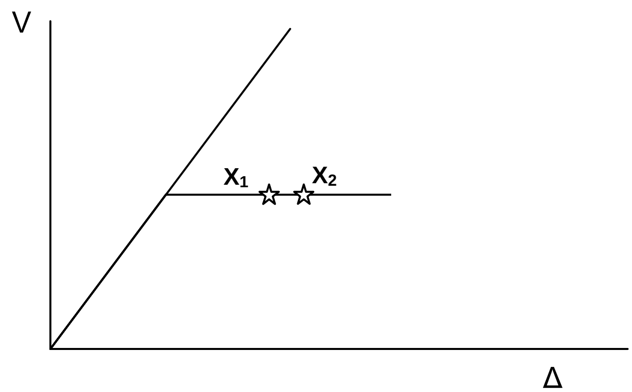
Since B<1.7, this building is not considered to be torsionally sensitive. In general buildings with the main force resisting elements located around the exterior of the building will not be torsionally sensitive.

10. Discussion
A couple of important issues related to this design example will be discussed in this section.

a) Why should the N-S walls be considered to resist entire torsional effects?
The distribution of forces to the various elements in the structure is generally based on the relative elastic stiffnesses of the elements, unless the diaphragms are considered to be flexible and then the forces are distributed on the basis of contributory masses. The present example structure with four floors of concrete construction can be considered as having rigid diaphragms, and an elastic analysis was performed to determine the wall forces due to the torsional effects. Because the N-S walls are so much longer and stiffer than the E-W walls, and more widely separated, it is expected that they will resist most of the torque from the eccentricity. However, since we are designing the structures to respond inelastically, the distribution of forces from an elastic analysis should always be questioned. An argument is presented below to show that if

the forces in the E-W walls are designed to be equal, they will not contribute to the torsional resistance.

The elastic torsional analysis for the forces in the E-W direction result in additional forces of ±154 kN in the E-W walls and ±1070 kN in the N-S walls (see Table 4). If all the torque is resisted by the N-S walls, the force in these walls would be ±1120 kN (an increase of only 50 kN).



For the earthquake load in the E-W direction the E-W walls must resist the total base shear in this direction and so they will have reached their yield strength and progressed along the flat portion of the shear/displacement curve as shown in the figure (assuming they have equal strength). The torsional load will have caused a small rotation of the diaphragms and so wall X_2 will have a slightly larger displacement than wall X_1, as shown on the figure. Had the walls remained elastic, the shear in wall X_2 would then be greater than wall X_1 and this would contribute to the torsional resistance. However in the nonlinear case, they both have the same shear resistance and so do not contribute to the torsional resistance. Thus in this example, all the torsion should be resisted by the longer N-S walls. The N-S walls are designed to resist the loads in the N-S direction but also to provide the torsional resistance from the loads in the E-W direction. However, it is highly unlikely that the maximum forces in the N-S walls from the two directions would occur at the same time, and practice has been to consider only 30% of the loads in one direction when combining with the loads in the other direction. Thus the forces in the N-S walls at the time of the maximum torsional forces from the N-S direction could reach the yield level on one side, but the torsional displacement on the other side would be in the opposite direction, so the wall force would be much reduced in the other direction. The two N-S forces then provide a torque to resist the torsional motion. Although this resisting torque may not be as large as the elastic analysis would predict, the result would not be failure, but only slightly larger torsional displacements.

b) Application of the "100%+30%" rule

In the calculation of total wall seismic forces including the torsional effects (see step 7 above), the effect of seismic loads in E-W direction only was taken into consideration when calculating the forces in E-W walls. However, it is a good practice to consider the "100+30%" rule that requires the forces in any element that arise from 100% of the loads in one direction be combined with 30% of the loads in the orthogonal direction (for more details refer to NBCC 4.1.8.8.(1)c and the commentary portion in Section 1.5.9.3 of this document).

Let us determine the forces in one of the E-W walls, e.g. wall X_2, by applying the "100+30%"rule. If only 100% of the force in the E-W direction is considered, the total force in the wall is equal to (see Table 4):

$$V_{X2}^{E-W} = V_{X2o} + V_{X2t} = 1450 + 154 = 1604 kN$$

If the seismic load is applied in the N-S direction, the torsional moment would be determined based on the accidental eccentricity e_a (since the building is symmetrical in that direction), and so the torsional force in the wall X_2 can be prorated by the ratio of torsional eccentricities in the E-W and N-S directions as follows,

$$V_{X2}^{N-S} = V_{X2t} * \frac{e_a}{e_y} = 154 * \frac{2.0m}{7.73m} = 39.8 \approx 40kN$$

The total seismic force in the wall X_2 due to 100% of the load in E-W direction and 30% of the load in the N-S direction can be determined as

$$V_{X2} = V_{X2}^{E-W} + 0.3V_{X2}^{N-S} = 1604 + 0.3 * 40 = 1616kN$$

It can be concluded that the difference between the force of 1616 kN (when the “100+30%” rule is applied) and the force of 1604 kN (when the rule is ignored) is insignificant.

However, it can be shown that the “100+30%” rule would significantly influence the forces in the N-S walls. When the seismic force acts in the E-W direction, the force in the N-S wall (e.g. wall Y_1) due to torsional effects is equal to (see Table 4)

$$V_{Y1}^{E-W} = 1070kN$$

When the seismic force acts in the N-S direction, the total force in the wall Y_1 (including the effect of accidental torsion) can be determined as (see Example 1 for a detailed discussion on accidental torsion)

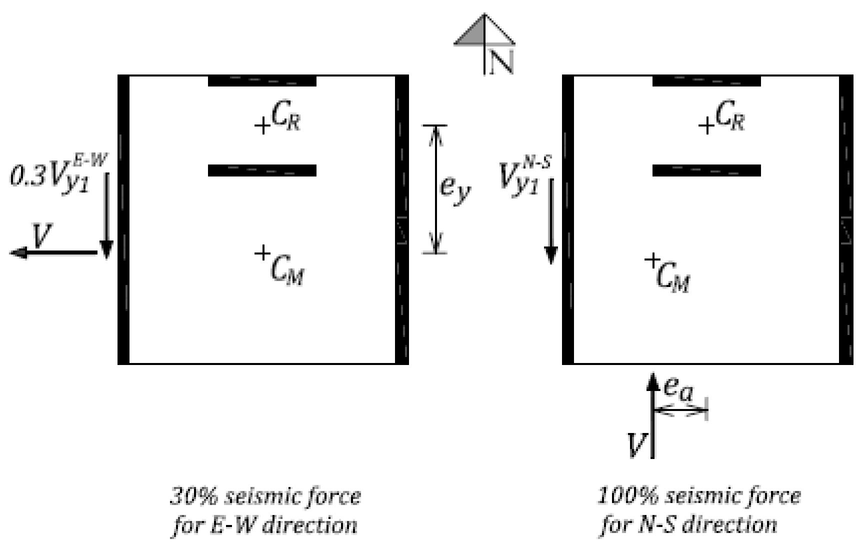
$$V_{Y1}^{N-S} = 0.6 * V = 0.6 * 2900 = 1740kN$$

So, if we apply the “100+30%” rule to 100% of the force in the N-S direction and 30% of the force in the E-W direction the resulting total force is equal to

$$V_{Y1} = V_{Y1}^{N-S} + 0.3V_{Y1}^{E-W} = 1740 + 0.3 * 1070 = 2061kN$$

In this case, it can be concluded that the difference between the force of 2061 kN (when the “100+30%” rule is applied) and the force of 1740 kN (when the rule is ignored) is significant (around 18%). This is illustrated on the figure below.

For those cases where there is a large eccentricity in one direction and the torsional forces are mainly resisted by elements in the other direction, the contribution from the “100+30%” rule can be significant.



<u>EXAMPLE 3</u>: Seismic load distribution in a masonry building considering both rigid and flexible diaphragm alternatives

Consider a single-storey commercial building located in Nanaimo, BC on a Class C site. The building plan and relevant elevations are shown on the figure below. The building has an open north-west façade consisting mostly of glazing. The roof elevation is at 4.8 m above the foundation. The roof structure is supported by 240 mm reinforced block masonry walls and steel columns on the north-west side. Masonry properties should be determined based on 20 MPa block strength and Type S mortar (use f'_m of 10.0 MPa). Grade 400 steel has been used for the reinforcement.

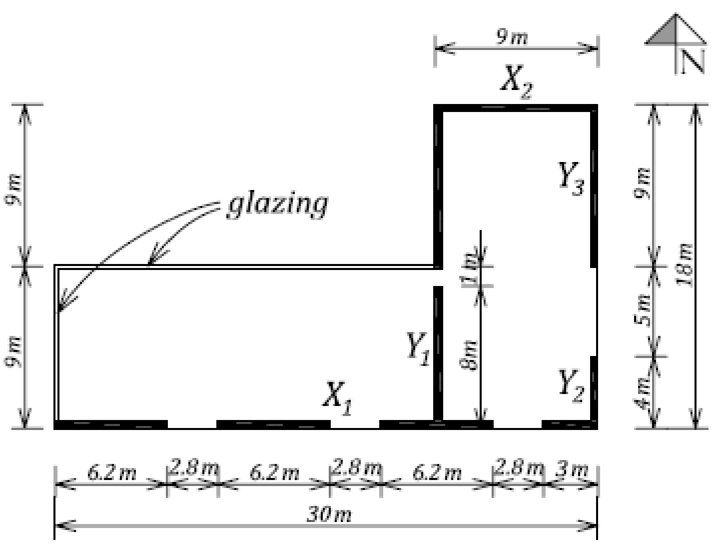
Masonry walls should be treated as “conventional construction” according to NBCC 2005 and CSA S304.1. A preliminary seismic design has shown that the total seismic base shear force for the building is equal to $V = 700$ kN. This force was determined based on the total seismic weight W of 2340 kN and the seismic coefficient equal to 0.3, that is, $V = 0.3W$.

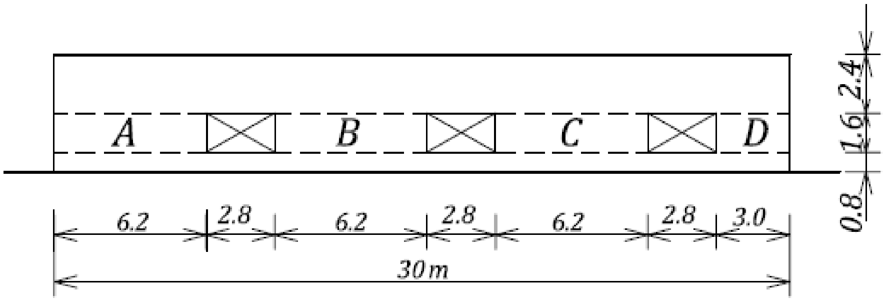
This example will determine the seismic forces in the N-S walls (Y_1 to Y_3) due to seismic force acting in the N-S direction for the following two cases:
a) Rigid roof diaphragm (consider torsional effects), and
b) Flexible roof diaphragm.

Finally, the wall forces obtained in parts a) and b) will be compared and the differences will be discussed.

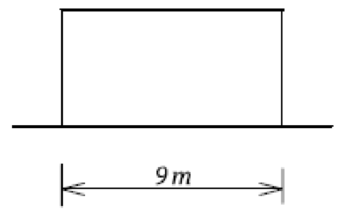
Note that both flexible and rigid diaphragms are considered to have the same weight, although this would be unlikely in a real design application. Also, the columns located on the north-west side are neglected in the seismic design calculations.

Specified loads:
roof = 3.5 kPa
25% snow load = 0.6 kPa
wall weight = 5.38 kPa (240 mm blocks solid grouted; this is a conservative assumption)

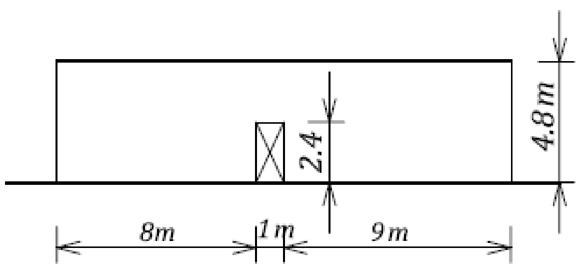




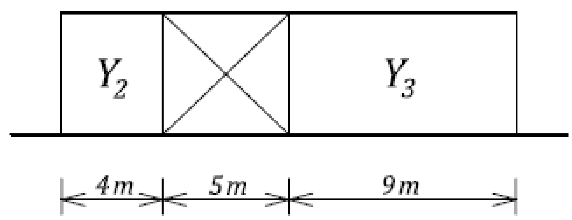
Wall X_1



Wall X_2



Wall Y_1



Walls Y_2 & Y_3

SOLUTION:

a) Rigid diaphragm

Torsional moment (torque) is a product of the seismic force and the eccentricity between the centre of resistance (C_R) and the centre of mass (C_M). The coordinates of the centre of mass will be determined taking into account the influence of wall masses, the upper half of which are supported laterally by the roof. The calculations are summarized in Table 1 below. Note that the centroid of the roof area is determined by dividing the roof plan into two rectangular sections.

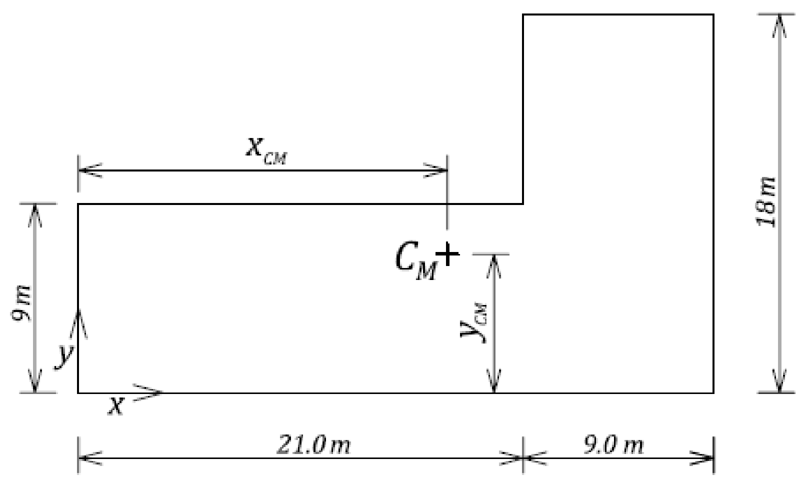
Table 1. Calculation of the Centre of Mass (C_M)

Wall	W_i (kN)	X_i (m)	Y_i (m)	$W_i * X_i$	$W_i * Y_i$
X1	387	15.00	0.00	5810	0
X2	116	25.50	18.00	2963	2092
Y1	232	21.00	9.00	4880	2092
Y2	52	30.00	2.00	1548	103
Y3	116	30.00	13.50	3486	1569
Roof 1	1107	15.00	4.50	16605	4982
Roof 2	332	25.50	13.50	8466	4482
	2343			43759	15319

The C_M coordinates have been determined from the table as follows (see the figure below):

$$x_{CM} = \frac{\sum_i W_i * X_i}{\sum_i W_i} = \frac{43757.02}{2343.86} = 18.68 \text{ m}$$

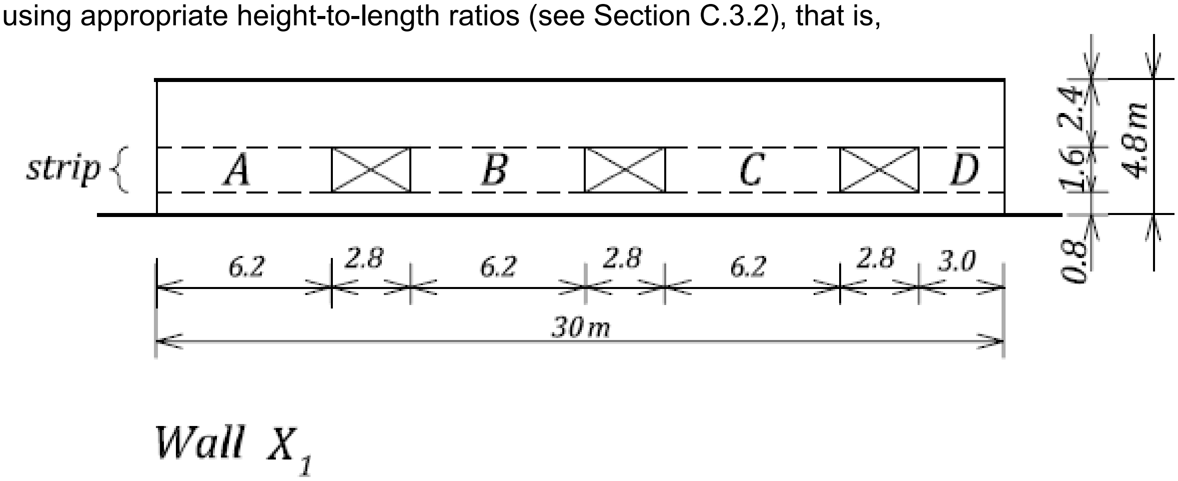
$$y_{CM} = \frac{\sum_i W_i * Y_i}{\sum_i W_i} = \frac{15324.38}{2343.86} = 6.54 \text{ m}$$



Next, the coordinates of the centre of resistance (C_R) will be determined. Wall X_1 has several openings and the overall wall stiffness is determined using the method explained in Section C.3.3 by considering the deflections of the following components for a unit load (see the figure on the next page):

- solid wall with 4.8 m height and 30 m length – cantilever (Δ_{solid})
- an interior strip with 1.6 m height (equal to the opening height) and 30 m length – cantilever (Δ_{strip})
- piers A, B, C, and D – cantilevered (Δ_{ABCD}) (the stiffness of the piers A, B, C, and D is summed and the inverse taken as Δ_{ABCD})

The stiffness of each component is based on the following equation for the cantilever model by using appropriate height-to-length ratios (see Section C.3.2), that is,



$$\frac{K}{E_m * t} = \frac{1}{\left(\frac{h}{l}\right)\left[4\left(\frac{h}{l}\right)^2 + 3\right]}$$

The overall wall deflection is determined from the combined pier deflections, as follows:

$$\Delta_{X1} = \Delta_{solid} - \Delta_{strip} + \Delta_{ABCD}$$

Note that the strip deflection is subtracted from the solid wall deflections - this removes the entire portion of the wall containing all the openings, which is then replaced with the deflection of the four piers.

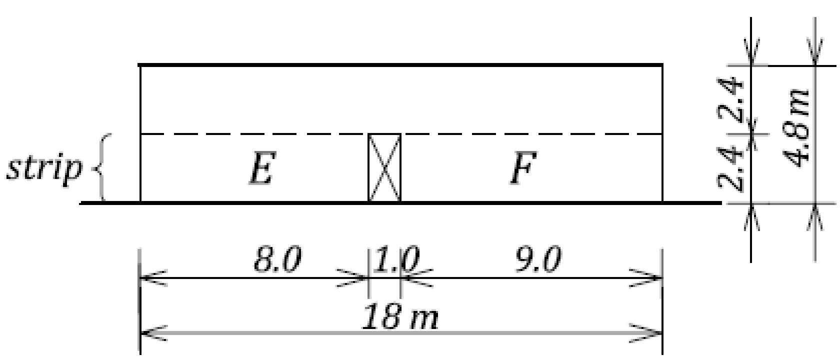
Finally, the stiffness of the wall X_1 is equal to the reciprocal of the deflection (see Table 2), as follows

$$K_{X1} = \frac{1}{\Delta_{X1}} = 1.71$$

Table 2. Wall X_1 Stiffness Calculations

Wall	t (m)	h (m)	l (m)	**End conditions**	h/l	$K/(E*t)$	**Displacement**	$K_{final}/(E*t)$
Solid	0.24	4.8	30.0	cant	0.160	2.015	0.496	
Opening strip	0.24	1.6	30.0	cant	0.053	6.226	-0.161	
X1A	0.24	1.6	6.2	cant	0.258	1.186		
X1B	0.24	1.6	6.2	cant	0.258	1.186		
X1C	0.24	1.6	6.2	cant	0.258	1.186		
X1D	0.24	1.6	3.0	cant	0.533	0.453		
					$\sum$ (ABCD)	4.012	0.249	
							0.585	1.709

The stiffness of wall Y_1 is determined in the same manner (see the figure below). The calculations are summarized in Table 3.



Wall Y_1

Table 3. Wall Y_1 Stiffness Calculations

Wall	t **(m)**	h **(m)**	l **(m)**	**End conditions**	h/l	$K/(E*t)$	**Displacement**	$K_{final}/(E*t)$
Solid	0.24	4.8	18	cant	0.267	1.142	0.876	
Opening strip	0.24	2.4	18	cant	0.133	2.442	-0.409	
Pier E	0.24	2.4	8	cant	0.300	0.992		
Pier F	0.24	2.4	9	cant	0.267	1.142		
					sum(EF)	2.134	0.469	
							0.935	1.070

Next, the centre of resistance (C_R) will be determined, and the calculations are presented in Table 4.

Table 4. Calculation of the Centre of Resistance (C_R)

Wall	t (m)	h (m)	l (m)	End cond.	h/l	$\frac{K}{E*t}$	K_x (kN/m)	K_y (kN/m)	X_i (m)	Y_i (m)	$K_y * X_i$	$K_x * Y_i$
X1	0.24					1.709*	3.49E+06	0	15	0		0.00E+00
X2	0.24	4.8	9	cant	0.53	0.453	9.24E+05	0	25.5	18		1.66E+07
Y1	0.24					1.070**	0	2.18E+06	21	0	4.58E+07	
Y2	0.24	4.8	4	cant	1.20	0.095	0	1.94E+05	30	0	5.82E+06	
Y3	0.24	4.8	9	cant	0.53	0.453	0	9.24E+05	30	0	2.77E+07	
							4.41E+06	3.30E+06			7.94E+07	1.66E+07

Notes:
* - see Table 2
** - see Table 3

Note that all walls and piers in this example were modeled as cantilevers (fixed at the base and free at the top). For more discussion related to modelling of masonry walls and piers for seismic loads see Section C.3. The modulus of elasticity for masonry is taken as $E_m = 8.5*10^6$ kPa (corresponding to f'_m of 10 MPa).

The C_R coordinates can be determined as follows (see the figure below):

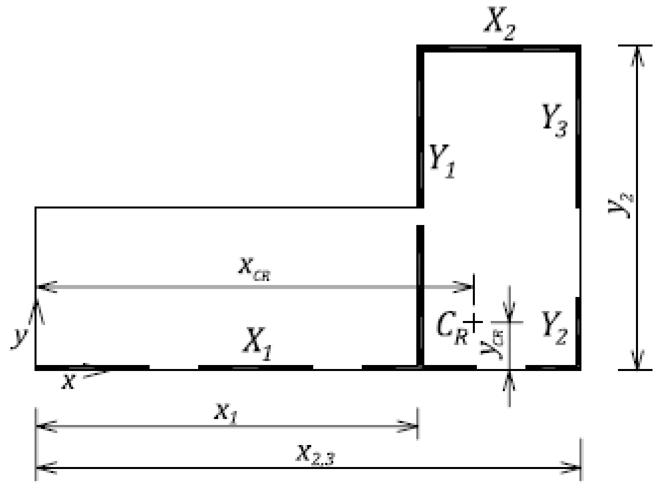
$$x_{CR} = \frac{\sum_i K_{yi} * x_i}{\sum_i K_{yi}} = \frac{7.94*10^7}{3.30*10^6} = 24.05 \text{ m}$$

$$y_{CR} = \frac{\sum_i K_{xi} * y_i}{\sum_i K_{xi}} = \frac{1.66*10^7}{4.41*10^6} = 3.77 \text{ m}$$

Next, the eccentricity needs to be determined. Since we are considering the seismic load effects in the N-S direction, we need to determine the actual eccentricity in the x-direction (e_x), that is, $e_x = x_{CR} - x_{CM} = 24.05 - 18.68 = 5.37$ m

In addition, an accidental eccentricity needs to be considered, as follows:
$e_a = \pm 0.1 D_{nx} = \pm 0.1 * 30 = \pm 3.0$ m

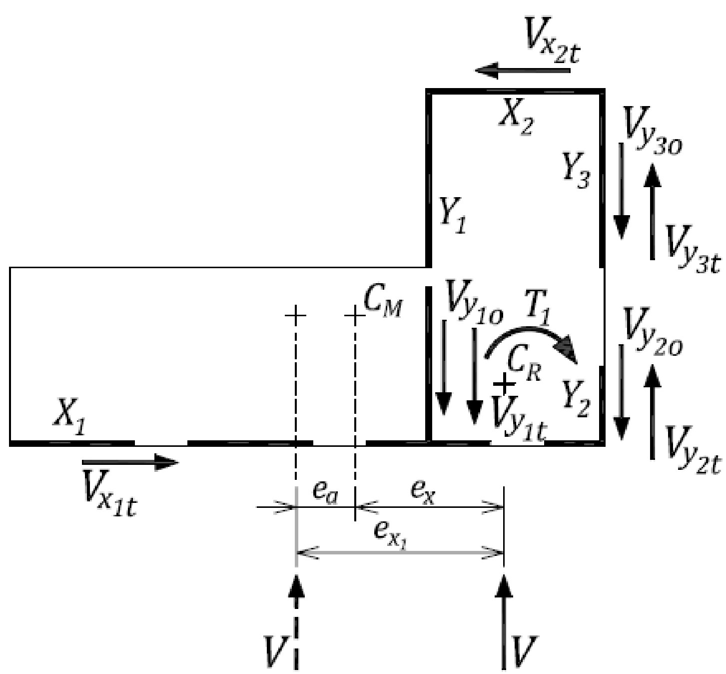
The total maximum eccentricity in the x-direction assumes the following two values depending on the sign of the accidental eccentricity, that is,
$e_{x1} = e_x + e_a = 5.37 + 3.0 = 8.37$ m
$e_{x2} = e_x - e_a = 5.37 - 3.0 = 2.37$ m



The torsional moment is determined as a product of the shear force and the eccentricity, that is,
$T_1 = V * e_{x1} = 700 * 8.37 \approx 5860$ kNm
$T_2 = V * e_{x2} = 700 * 2.37 \approx 1660$ kNm

The seismic force in each wall can be determined as the sum of the two components: translational (no torsional effects) and torsional, that is,
$V_i = V_{io} + V_{it}$
where
$V_{io} = V * \frac{K_i}{\sum K_i}$ translational component
$V_{it} = \frac{T * c_i}{J} * K_i$ torsional component
$J = \sum K_{xi} \cdot c_{xi}^2 + \sum K_{yi} \cdot c_{yi}^2 = 2.97 * 10^8$ torsional rigidity (see Table 5)
c_{xi}, c_{yi} - distance of the wall centroid from the centre of resistance (C_R)

The calculation of translational and torsional forces is presented in Table 5. Translational and torsional force components due to the eccentricity e_{x1} and the torsional moment T_1 are shown on the figure. Note that the torque T_1 causes rotation in the same direction like the force V (showed by the dashed line) around point C_R (this is illustrated on Figure 1-18). The wall forces shown on the diagram are in the directions to resist the shear V and torque T_1, thus on wall Y1 the translational



force and torsional force act in the same direction, while in walls Y2 and Y3 these forces act in the opposite direction. The calculation of the forces is presented in Table 5 where the sign convention has horizontal wall forces positive to the left and vertical forces positive down, resulting in negative values for the torsional forces in walls X1, Y2 and Y3.

Table 5. Seismic Shear Forces in the Walls due to Seismic Load in the N-S Direction

Wall	K_i (kN/m)	c_i (m)	$K_i * c_i^2$	$K_y / \sum K_y$	V_o (kN)	V_{1t} (kN)	V_{1total} (kN)	V_{2t} (kN)	V_{2total} (kN)	V_{govern} (kN)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
X1	3.49E+06	-3.77	4.96E+07			-260	-260	-74	-74	260
X2	9.24E+05	14.23	1.87E+08			260	260	74	74	260
$\sum K_x$	4.41E+06									
Y1	2.18E+06	3.05	2.03E+07	0.66	463	131	594	37	500	594
Y2	1.94E+05	-5.95	6.87E+06	0.06	41	-23	18	-6	35	35
Y3	9.24E+05	-5.95	3.27E+07	0.28	196	-109	87	-31	165	165
$\sum K_y$	3.30E+06			1.00	700					
		$\sum K_i * c_i^2$	2.97E+08							

It should be noted that there are two total seismic forces for each wall in the N-S direction (corresponding to torsional moments T_1 and T_2) – see columns (8) and (10) in Table 5. The governing force to be used for design is equal to the larger of these two forces, as shown in column (11) of Table 5. Note that, in some cases, torsional forces have a negative sign and cause a reduction in the total seismic force, like in the case of walls Y2 and Y3.

b) Flexible diaphragm

It is assumed in this example that flexible diaphragms are not capable of transferring significant torsional forces to the walls perpendicular to the direction of the inertia forces. Therefore, the wall forces are determined as diaphragm reactions, assuming that diaphragms D1 and D2 act as beams spanning between the walls, as shown on the figure below. The diaphragm loads include the inertia loads of the walls supported laterally by the diaphragm. The SFRS wall inertia forces are added to the forces supporting the diaphragms to get the total wall load. The seismic coefficient of 0.3 will be used in these calculations (as defined at the beginning of this example).

Shear forces in the walls Y_{1a} and Y_2 (diaphragm D1):
Seismic force in the diaphragm D1 is due to the roof seismic weight and the wall X_1 inertia load:

$$V_{D1} = 0.3*[(9m*30m)*(3.5kPa+0.6kPa)+2.4m*30m*5.38kPa] = 448kN$$

The diaphragm is considered as a beam with the reactions at the locations of walls Y_{1a} and Y_2, that is,

$$R_{Y1a} = 448kN*15m/9m = 747kN$$

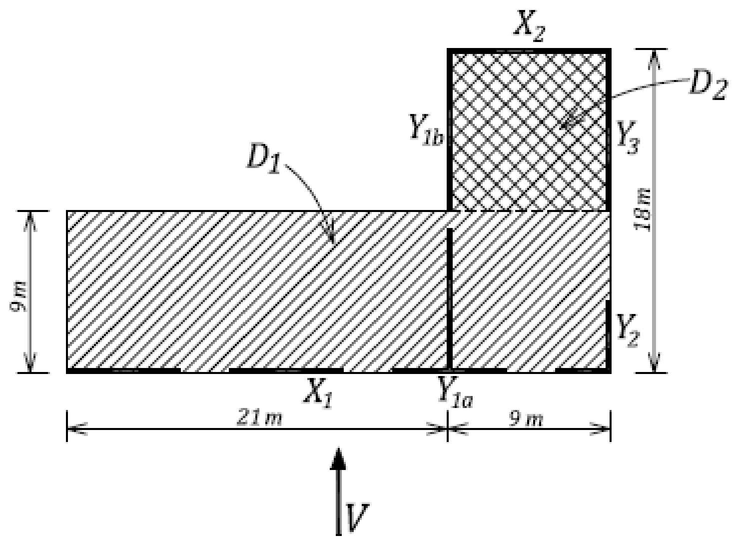
and

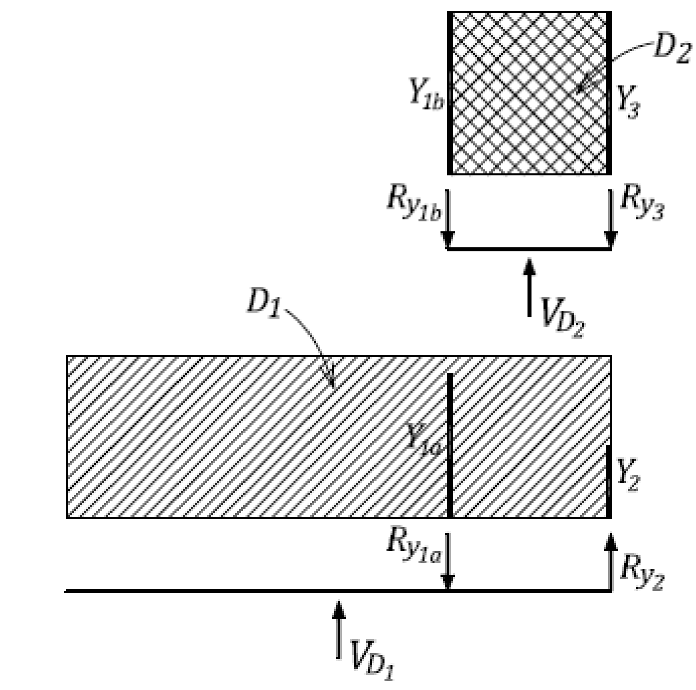
$R_{Y2} = V_{D1} - R_{Y1a} = 448 - 747 = -299kN$ (opposite direction from R_{Y1a} is required to satisfy equilibrium)

The total force in each wall is obtained when the wall inertia load is added to the diaphragm reaction, that is,

$V_{Y1a} = R_{Y1a} + V_w = 747 + 0.3 * 2.4m * 9m * 5.38kPa = 782kN$

$V_{Y2} = R_{Y2} + V_w = -299 + 0.3 * 2.4m * 4m * 5.38kPa = -284kN$ (note: this force has opposite direction from force V_{Y1a})





<u>Shear forces in the walls Y_{1b} and Y_3 (diaphragm D2):</u>

Seismic force in the diaphragm D2 is due to the roof seismic weight and the wall X_2 inertia load:

$V_{D2} = 0.3 * \left[(9m * 9m) * (3.5kPa + 0.6kPa) + 2.4m * 9m * 5.38kPa\right] = 134.5kN$

The diaphragm is considered as a beam with the reactions at the locations of walls Y_{1b} and Y_3, that is,

$R_{Y1b} = R_{Y3} = 134.5/2 = 67.3kN$

The total force in each wall is obtained when the wall inertia load is added to the diaphragm reaction, that is,

$V_{Y1b} = R_{Y1b} + V_w = 67 + 0.3 * 2.4m * 9m * 5.38kPa = 102kN$

$V_{Y3} = R_{Y3} + V_w = 67 + 0.3 * 2.4m * 9m * 5.38kPa = 102kN$

Total shear force in wall Y_1:

The total seismic force in the wall Y_1 is equal to

$V_{Y1} = V_{Y1a} + V_{Y1b} = 782 + 102 = 884kN$

Shear forces in walls Y_2 and Y_3:

The total shear force in the combined walls Y_2 and Y_3 is equal to

$V_{Y23} = V_{Y2} + V_{Y3} = -284 + 102 = -182kN$

This force will then be distributed to these walls in proportion to the wall stiffness, as follows (the wall stiffnesses are presented in Table 4):

$$V_{Y2} = \frac{K_{Y2}}{K_{Y2} + K_{Y3}} * V_{Y23} = \frac{1.94 * 10^5}{1.94 * 10^5 + 9.24 * 10^5} * (-182) = 0.17 * (-182) = -32kN$$

$V_{Y3} = V_{Y23} - V_{Y2} = -182 - (-32) = -150kN$

The comparison

Shear forces in the walls Y_1 to Y_3 obtained in parts a) and b) of this example are summarized on the figure below. A comparison of the shear forces is presented in Table 6.

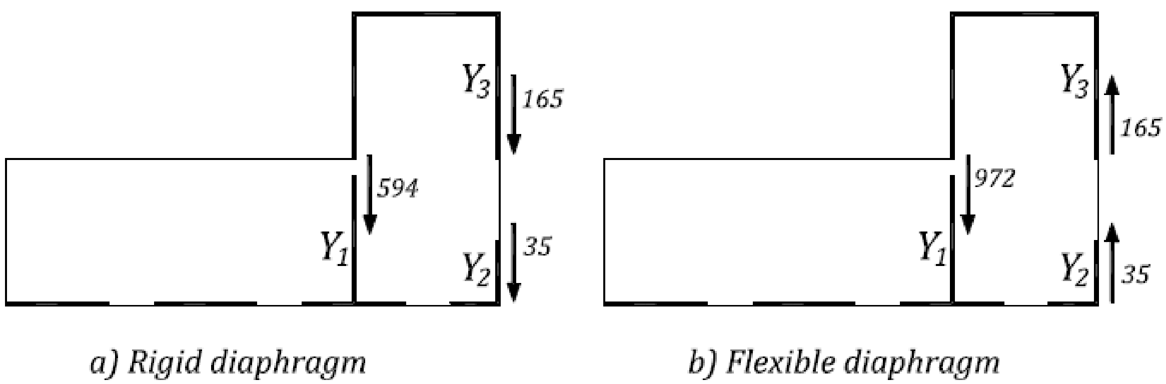


Table 6. Shear Forces in the Walls Y_1 to Y_3 for Rigid and Flexible Diaphragms

Wall	Shear forces (kN)	
	Rigid diaphragm (part a)	Flexible diaphragm (part b)
Y_1	594	972 (884)
Y_2	35	35 (32)
Y_3	165	165 (150)

Note that, for the flexible diaphragm case, values in the brackets are actual forces. These values are increased by 10 % to account for accidental eccentricity.

It can be observed from the table that the flexible diaphragm assumption results in the same seismic forces for the walls Y_2 and Y_3, and an increase in the wall Y_1 force.

Deflection calculations

A fundamental question related to diaphragm design is: when should a diaphragm be modeled as a rigid or a flexible one? This is discussed in Section 1.5.9.4. A possible way for comparing the extent of diaphragm flexibility is through deflections. The deflection calculations for the rigid and flexible diaphragm case are presented below.

- ***Rigid diaphragm (see Example 2, step 8 for a similar calculation)***

The deflection will be calculated for point A as this should be the maximum. First, a reduction in the wall stiffness to account for the effect of cracking will be determined following the approach presented in Section C.3.5. The reduced stiffness will be determined for wall Y_2 according to equation (15) from Section C.3.5, that is,

$$K_{ce} = (\frac{100}{f_y} + \frac{P_f}{f'_m A_e})K_c$$

where

K_c is the elastic uncracked stiffness

$P_f = 9.0*(9.0/2)*3.5 = 142$ kN (axial force due to dead load in wall X_2)

$A_e = (240*10^3)*9.0 = 216*10^4$ mm^2 (effective cross sectional area for 240 mm block wall, solid grouted, length 9.0 m; see Table D-1 for A_e values for the unit wall length)

f'_m =10.0 MPa

$f_y = 400$ MPa (Grade 400 steel)

thus

$$K_{ce} = (\frac{100}{400} + \frac{142*10^3}{10.0*216*10^4})K_c = 0.26K_c$$

Next, the translational displacement at point A can be calculated as follows:

$$\Delta^A{}_0 = \frac{V}{0.26\sum K_Y} = \frac{700kN}{0.26*3.3*10^6 kN/m} = 0.82mm$$

Subsequently, the torsional displacement at point A will be determined. Torsional rotation of the building θ can be found from the following equation:

$$\theta = \frac{T}{J} = \frac{5860kNm}{0.26*297*10^6} = 7.59*10^{-5}\text{ rad}$$

where (see the torsional calculations performed in part a) of this example)

$T = 5860$ kNm torsional moment

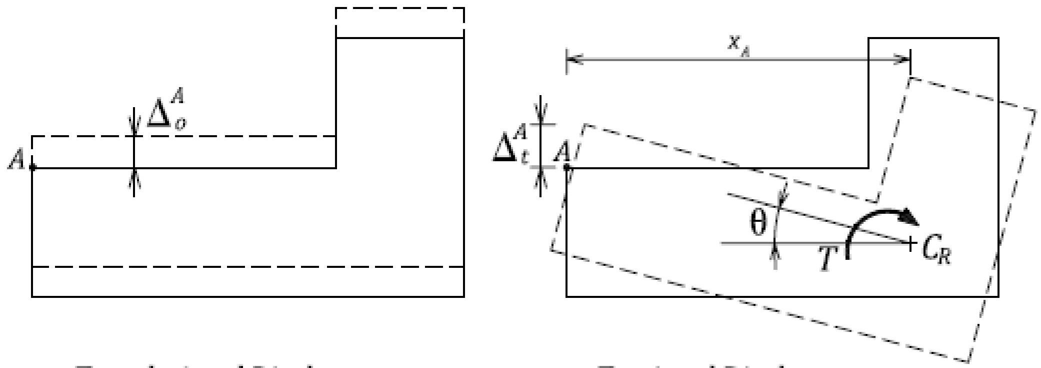
$J = 297*10^6$ elastic torsional stiffness (this value is reduced by 0.5 to take into account the cracking in the walls)

The torsional displacement at point A:

$$\Delta^A{}_t = \theta * x_A = 7.59*10^{-5}*24.05m = 1.82mm$$

The total displacement at point A is can be found as follows (note that the displacements need to be multiplied by $R_d R_o / I_E$ ratio, where I_E = 1.0):

$$\Delta^A{}_{\max} = \left(\Delta^A{}_0 + \Delta^A{}_t\right) * R_d R_o = (0.82 + 1.82) * 1.5 * 1.5 = 6.0mm$$



Translational Displacement *Torsional Displacement*

- **Flexible diaphragm**

As a first approximation the calculation will consider a 21 m long diaphragm portion as a cantilever beam subjected to the total shear force equal to:

$$V_D = 0.3 * [(9m * 21m) * (3.5kPa + 0.6kPa) + 2.4m * 21m * 5.38kPa] = 314kN$$

and the equivalent uniform load is equal to

$v_D = V_D / L = 15.0$ kN/m

where

$L = 21.0$ m diaphragm length for the cantilevered portion

The real deflection will be larger since the diaphragm acting as a cantilever is not fully fixed at the wall Y_1, and walls Y_1, Y_2, and Y_3 also deflect; both effects provide some rotation at the fixed end of the cantilever.

Consider a plywood diaphragm with the following properties:

$E = 1500$ MPa plywood modulus of elasticity

$G = 600$ MPa plywood shear modulus

$t_D = 25.4$ mm (1" plywood thickness)

$A = b * t_D = 9.0m * 0.0254m = 0.23$ m^2

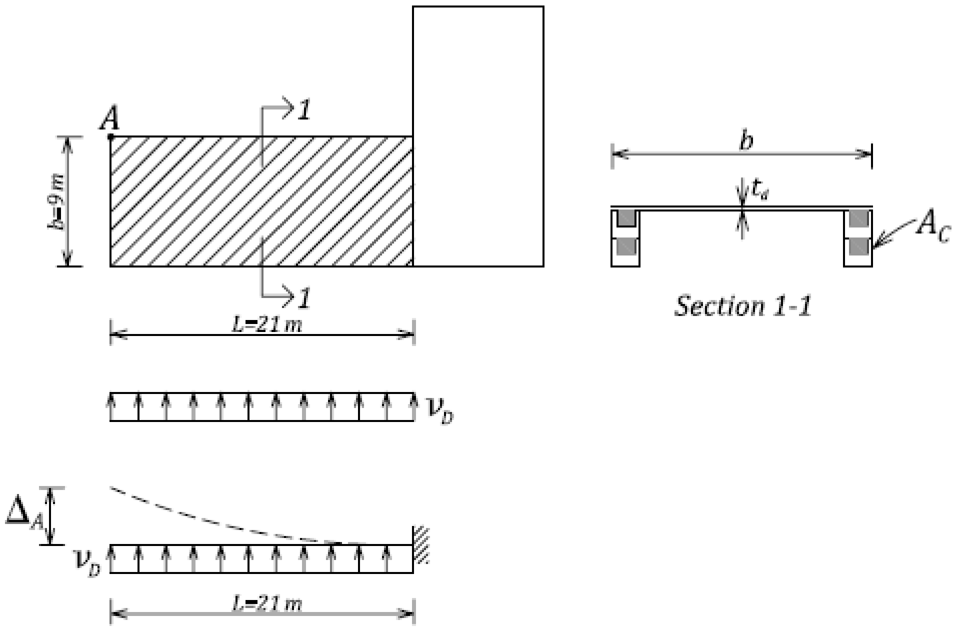
Let us assume that the two courses of grouted bond beam block act as a chord member, as shown on the figure. The roof-to-wall connection is achieved by means of nails driven into the anchor plate and hooked steel anchors welded to the plate embedded into the masonry. The corresponding moment of inertia around the centroid of the diaphragm can be found as follows:

$$I = 2 * A_c * \left(\frac{b}{2}\right)^2 = 2 * 0.096 * \left(\frac{9.0}{2}\right)^2 = 3.89 \text{ m}^4$$

where

$A_c = 2 * (0.24m * 0.2m) = 0.096$ m^2 chord area (two grouted 240 mm blocks)

$E_m = 8.5 * 10^6 kPa$ masonry modulus of elasticity based on f'_m = 10.0 MPa (solid grouted 20 MPa blocks and Type S mortar)



The total displacement at point A is equal to the combination of flexural and shear component, that is,

$$\Delta^A = \frac{v_D * L^4}{8E * I} + \frac{1.2V_D * L}{2 * A * G} = \frac{15.0 * (21.0)^4}{8 * 8.5 * 10^6 * 3.89} + \frac{1.2 * 314 * 21.0}{2 * 0.23 * 600 * 10^3} = (11.0 + 29.0) * 10^{-3} = 40 * 10^{-3}\,m = 40mm$$

The total displacement at point A is can be found by multiplying the above displacement by $R_d R_o / I_E$ ratio, that is,

$$\Delta^A{}_{\max} = \Delta^A * R_d R_o = 40 * 1.5 * 1.5 = 90mm$$

A quick check of the additional deflection caused by rotation at the fixed end of the cantilever indicates that an additional 50 mm could be expected at point A. Thus the total displacement would be about 140 mm.

By comparing the displacements for the rigid and flexible diaphragm model, it can be observed that the difference is significant:

$\Delta^A{}_{\max} = 6mm$ rigid diaphragm model

$\Delta^A{}_{\max} = 90mm$ flexible diaphragm model

Had the flexible diaphragm been used, the lateral drift ratio at point A would be equal to:

$$DR = \frac{\Delta_{\max}}{h_w} = \frac{90}{4800} = 0.019 = 1.9\ \%$$

The drift is within the NBCC 2005 limit of 2.5% (see Section 1.5.11); however, a flexible diaphragm would not be an ideal solution for this design – a rigid diaphragm would be the preferred solution.

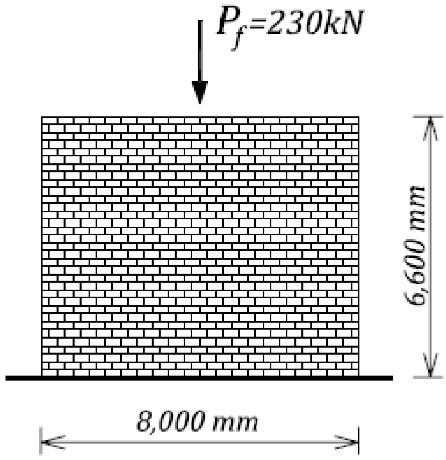
Discussion

In this example, seismic forces were determined for the N-S walls due to seismic load acting in the N-S direction. It should be noted, however, that there is a significant eccentricity causing torsional effects in the E-W walls due to seismic load acting in the E-W direction – these calculations were not included in this example.

EXAMPLE 4a: Minimum seismic reinforcement for a squat shear wall

Determine minimum seismic reinforcement according to CSA S304.1-04 for a loadbearing masonry shear wall located in an area with a seismic hazard index $I_E F_a S_a(0.2)$ of 0.66. The wall is subjected to axial dead load (including its own weight) of 230 kN.

Use 200 mm hollow concrete blocks of 15 MPa unit strength and Type S mortar. Grade 400 steel reinforcing bars (yield strength f_y = 400 MPa) and cold-drawn galvanized wire (ASWG) joint reinforcement are used for this design.



Wall dimensions:
l_w =8000 mm length
h_w = 6600 mm height
t = 190 mm thickness

SOLUTION:

The purpose of this example is to demonstrate how the minimum seismic reinforcement area should be determined and distributed in horizontal and vertical direction. Once the reinforcement has been selected in terms of its area and distribution, the flexural and shear resistance of the wall will be determined and the capacity design issues discussed, as well as the seismic safety implications of vertical and horizontal reinforcement distribution.

1. Material properties

Steel (both reinforcing bars and joint reinforcement):

f_y = 400 MPa ϕ_s = 0.85

Note that the cold-drawn galvanized wire has higher yield strength than Grade 400 steel, but it will be ignored for the small area included.

Masonry:

ϕ_m = 0.6

Assume partially grouted masonry. For 15MPa blocks and Type S mortar, it follows from Table 4 of S304.1-04 that

f'_m = 9.8 MPa

Based on Note 3 to Table 4, this f'_m value is normally used for hollow block masonry but can also be used for partially grouted masonry if the grouted area is not considered.

2. Find the minimum seismic reinforcement area and spacing (see Section 2.5.4.7 and Table 2-2).

Since $I_E F_a S_a(0.2)$=0.66 > 0.35, minimum seismic reinforcement must be provided (S304.1 Cl.10.15.2.2).

Seismic reinforcement area

Loadbearing walls, including shear walls, shall be reinforced horizontally and vertically with steel having a minimum area of

$A_{s\min} = 0.002A_g$ = 0.002*(190*10^3 mm^2/m) = 380 mm^2/m

for 190 mm block walls, where

A_g =(1000mm)*(190mm)=190*10^3 mm^2/m gross cross-sectional area for a unit wall length of 1 m

Minimum area in each direction (one-third of the total area):

$A'_{h\min} = A'_{v\min} = 0.00067A_g = \frac{A_{s\min}}{3} = \frac{380}{3} = 127$ mm^2/m

Thus the minimum total vertical reinforcement area

$A_{v\min} = 127 * l_w$ = (127 mm^2/m)(8 m) = 1016 mm^2

In distributing seismic reinforcement, the designer may be faced with the dilemma: should more reinforcement be placed in the vertical or in the horizontal direction? In theory, 1/3rd of the total amount of reinforcement can be placed in one direction and the remainder in the other direction. In this example, less reinforcement will be placed in the vertical direction, and more in the horizontal direction. The rationale for this decision will be explained later in this example.

Vertical reinforcement (area and distribution) (see Table 2-2):

According to S304.1 Cl.10.16.4.3.2, spacing of vertical reinforcing bars shall not exceed the lesser of:

- $6(t+10)$ =6(190+10)=1200 mm
- 1200 mm
- $l_w/4$ =8000/4=2000 mm

Therefore, the maximum permitted spacing of vertical reinforcement is equal to $s = 1200$ mm.

Since the maximum permitted bar spacing is 1200 mm, a minimum of 8 bars are required (note that the total wall length is 8000 mm). Therefore, let us use 8-15M bars, so

A_v = 8*200 =1600 mm^2

(note that the resulting reinforcement spacing is going to be less than 1200 mm, which is the upper limit prescribed by CSA S304.1).

The corresponding vertical reinforcement area per metre length is

$A'_v = \frac{A_v}{l_w} * 1000$ = 200 mm^2/m > $A'_{v\min}$ =127 mm^2/m OK

Horizontal reinforcement (area and distribution) (see Table 2-2):
Let us consider a combination of joint reinforcement and bond beam reinforcement. According to S304.1 Cl.10.15.2.6, where both types of reinforcement are used, the maximum spacing of bond beams is 2400 mm and of joint reinforcement is 400 mm, so the following reinforcement arrangement is considered:

- 9 Ga. ladder reinforcement @ 400 mm spacing, and
- 2-15M bond beam reinforcement @ 2200 mm (1/3rd of the overall wall height). The area of ladder reinforcement (2 wires) is equal to $22.4 mm^2$, and the area of a 15M bar is $200 mm^2$. So, the total area of horizontal reinforcement per metre of wall height is

$$A'_h = \left(\frac{22.4}{400} + \frac{400}{2200}\right) * 1000 = 238 \text{ mm}^2/\text{m} > A'_{h\min} = 127 \text{ mm}^2/\text{m} \quad \text{OK}$$

So, the total area of horizontal and vertical reinforcement is

$$A_s = A'_v + A'_h = 200 + 238 = 438 \text{ mm}^2/\text{m} > A_{s\min} = 380 \text{ mm}^2/\text{m} \quad \text{OK}$$

Note that the total area ($438 mm^2/m$) exceeds the S304.1 minimum requirements ($380 mm^2/m$) by about 10%. It is difficult to select reinforcement that exactly meets the requirements, and also a reserve in reinforcement area provides additional safety for seismic effects.

3. Check whether the vertical reinforcement meets the minimum requirements for loadbearing walls (S304.1 Cl. 10.15.1.1 – see Table 2-2).
Since this is a shear wall, but also a loadbearing wall, pertinent reinforcement requirements would need to be checked, however the check is omitted from this example since it does not govern in seismic zones.

4. Determine the flexural resistance of the wall section (see Section C.1.1.2).
Design for combined effects of axial load and flexure will be performed by considering uniformly distributed vertical reinforcement. Based on the above discussion, the total area of vertical reinforcement is
A_{vt} = 1600 mm^2

At the base the wall is subjected to axial load P_f = 230 kN.

The in-plane moment resistance for the wall section can be determined approximately from the following equations:

$$\alpha_1 = 0.85 \qquad \beta_1 = 0.8$$

$$\omega = \frac{\phi_s f_y A_{vt}}{\phi_m f'_m l_w t} = \frac{0.85*400*1600}{0.6*9.8*8000*190} = 0.061$$

$$\alpha = \frac{P_f}{\phi_m f'_m l_w t} = \frac{230*10^3}{0.6*9.8*8000*190} = 0.026$$

$$c = \frac{\omega + \alpha}{2\omega + \alpha_1 \beta_1} l_w = \frac{0.061 + 0.026}{2*0.061 + 0.85*0.8}(8000) = 868 \text{ mm} \quad \text{neutral axis depth}$$

$$M_r = 0.5\phi_s f_y A_{vt} l_w \left(1 + \frac{P_f}{\phi_s f_y A_{vt}}\right)\left(1 - \frac{c}{l_w}\right) = 0.5 * 0.85 * \frac{400}{1000} * 1600 * \frac{8000}{1000}\left(1 + \frac{230 * 10^3}{0.85 * 400 * 1600}\right)\left(1 - \frac{868}{8000}\right)$$

$M_r = 2762$ kNm

5. Find the diagonal tension shear resistance (see Section 2.3.2 and CSA S304.1 Cl.10.10.1).

Find the masonry shear resistance (V_m):

$b_w = 190$ mm overall wall thickness

$d_v \approx 0.8 l_w = 6400$ mm effective wall depth

$\gamma_g = 0.5$ partially grouted wall

$P_d = 0.9 P_f$ = 207 kN

$v_m = 0.16(2 - \frac{M_f}{V_f d_v})\sqrt{f'_m}$ = 0.5 MPa

Note that a conservative assumption $\frac{M_f}{V_f d_v}$=1.0 has been made in the above equation.

$V_m = \phi_m (v_m b_w d_v + 0.25 P_d)\gamma_g$ = 0.6(0.5*190*6400+0.25*207*10^3)*0.5 = 198 kN

Steel shear resistance V_s:

$V_s = 0.6\phi_s \left(\sum A_v f_y \frac{d_v}{s}\right) = 0.6 * 0.85 * 608.8$ = 310 kN

where the shear reinforcement includes 9 Ga. joint reinforcement spaced at 400 mm, and 2-15M bond beam reinforcement at 2200 mm spacing, and so

$\sum A_v f_y \frac{d_v}{s} = \frac{22.4}{1000} * 400 * \frac{6400}{400} + \frac{400}{1000} * 400 * \frac{6400}{2200}$ = 608.8 kN

The total diagonal shear resistance is equal to

$V_r = V_m + V_s = 198 + 310 = 508$ kN

This is a squat shear wall because $\frac{h_w}{l_w} = \frac{6600}{8000} = 0.825 \le 1.0$.

Maximum shear allowed on the section is (S304.1 Cl.10.10.1.3):

$\max V_r = 0.4\phi_m \sqrt{f'_m} b_w d_v \gamma_g (2 - \frac{h_w}{l_w}) = 537$ kN

Since

$V_r < \max V_r$ OK

6. Sliding shear resistance (see Section 2.3.3)

The factored in-plane sliding shear resistance V_r is determined as follows.

μ = 1.0 for a masonry-to-masonry or masonry-to-roughened concrete sliding plane

A_s = 1600 mm^2 total area of vertical wall reinforcement

$T_y = \phi_s A_s f_y$ = 0.85*1600*400 = 544 kN

P_d = 207 kN

$P_2 = P_d + T_y$ = 207+544 = 751 kN

$V_r = \phi_m \mu P_2$ = 0.6*1.0*751=451 kN

7. Capacity design check (see Section 2.5.2)

At this point, both the moment resistance M_r and the diagonal shear resistance V_r for the wall section have been determined. Seismic design philosophy considers that it is desirable to design structural members such that the more ductile flexural failure takes place before the more brittle shear failure has been initiated. This is known as capacity design approach and is discussed in detail in Section 2.5.2.

In this case, the factored moment resistance is equal to

$M_r = 2762$ kNm

The nominal moment resistance can be estimated as follows

$$M_n = \frac{M_r}{\phi_s} = \frac{2762}{0.85} = 3249 \text{ kNm}$$

Shear force at the top of the wall that would cause the overturning moment equal to M_n is equal to

$$V_{nb} = \frac{M_n}{h_w} = \frac{3249}{6.6} = 492 \text{ kN}$$

To ensure that flexural failure takes place before the diagonal shear failure, it is required that (see Figure 2-22)

$V_{nb} \le V_r$

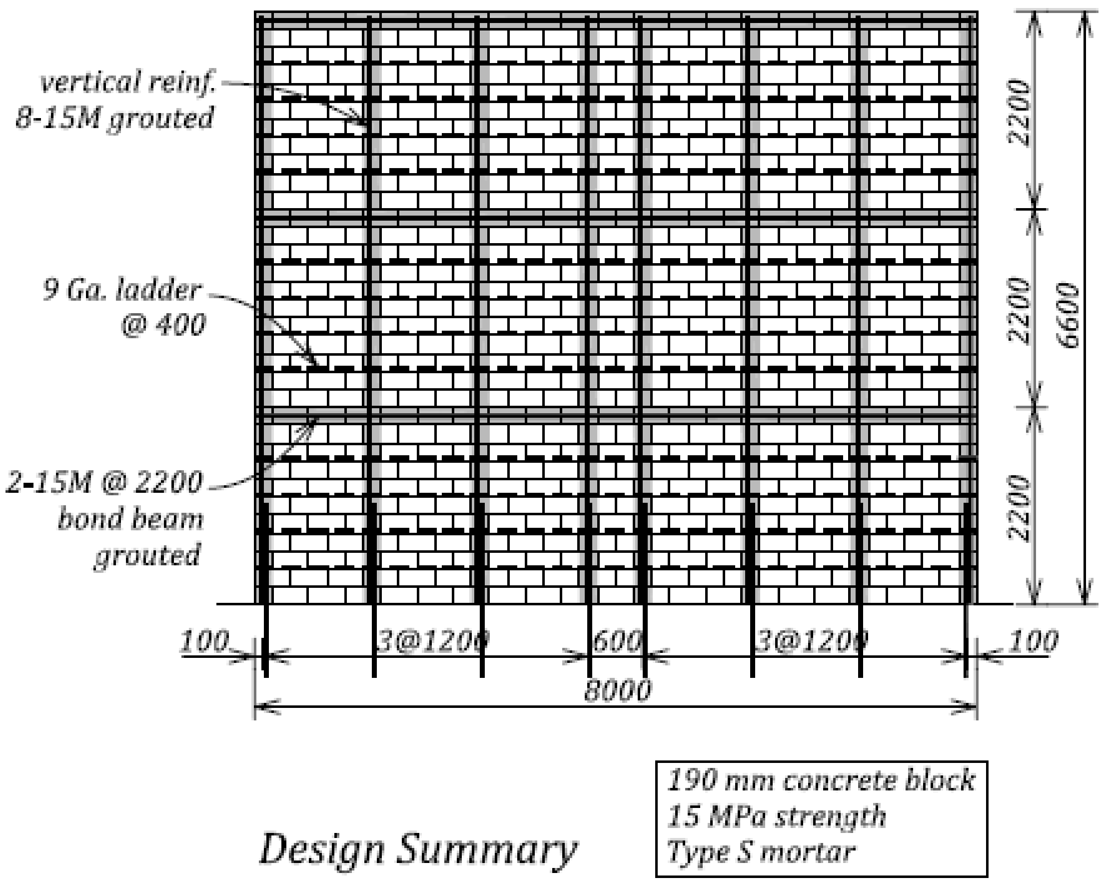
Since

$V_{nb} = 492 < V_r = 508$ kN

the capacity design criterion has been satisfied. It should be noted that CSA S304.1-04 does not formally require that capacity design approach be applied to all categories of reinforced masonry walls – it is mandatory only for "ductile walls" (limited ductility and moderately ductile shear walls). However, it is a good practice to consider the capacity design approach in designing all reinforced masonry walls in areas where seismic design is required by NBCC 2005 (note that this approach is followed in CSA A23.3 reinforced concrete design standard).

8. Design summary

The reinforcement arrangement for the wall under consideration is summarized below.



9. Discussion

It is important to consider all possible behaviour modes and identify the one that governs in this design. There are three shear forces:

a) $V_{nb} = 492$ kN shear force corresponding to flexural failure

b) $V_r = 508$ kN diagonal tension shear resistance

c) $V_r = 451$ kN sliding shear resistance

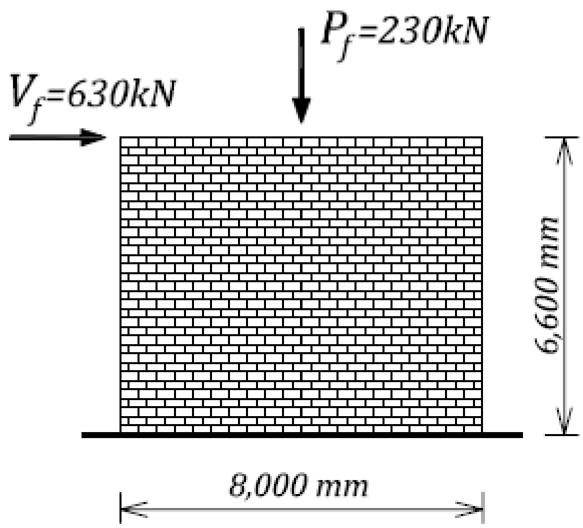
Since the sliding shear resistance value is the lowest, it can be concluded that the sliding shear mechanism is critical for this case, which is common for seismic design of squat shear walls.

As discussed at the beginning of this example, CSA S304.1 permits the minimum seismic reinforcement to be distributed in different ways. The solution presented above proposed that minimum reinforcement be placed in the vertical direction so that the capacity design criterion could be satisfied. Had the designer decided that more vertical reinforcement is required for meeting the flexural design requirements, (s)he could have used 10-15M bars instead of 8-15M bars (this would result in a 25% increase in the amount of vertical reinforcement). The corresponding amount of horizontal reinforcement could be reduced: 1-15M at 2200 mm spacing for bond beam reinforcement and 9 Ga. joint reinforcement at 400 mm spacing. This combination would meet the minimum seismic reinforcement requirements (250 mm^2/m vertical reinforcement and 146 mm^2/m horizontal reinforcement, with a total of 397 mm^2/m). For this arrangement of reinforcing the moment and shear resistance would be: $M_r = 3195$ kNm (a 16% increase compared to the previous value $M_r = 2762$ kNm) and $V_{nb} = 571$ kN shear force corresponding to flexural failure; however, $V_r = 392$ kN (diagonal tension shear resistance). Since $V_{nb} > V_r$ the capacity design criterion would not be met. As a result, this wall would be expected to fail in a shear (diagonal tension) mode characterized by a brittle failure, which is undesirable. Alternatively, the wall might fail in shear sliding mode, which is more desirable than the diagonal shear failure and often governs in low-rise reinforced masonry shear walls.

EXAMPLE 4b: Seismic design of a squat shear wall of $\boxed{\text{conventional}}$ construction

Design a single-storey squat concrete block shear wall shown in the figure below according to NBCC 2005 and CSA S304.1 seismic requirements for conventional construction. The building site is located in Ottawa, ON on Site Class C soil, and the seismic hazard index $I_E F_a S_a(0.2)$ is 0.66. The wall is subjected to a total dead load of 230 kN (including the wall self-weight) and an in-plane seismic force of 630 kN. Consider the wall to be solid grouted. Neglect the out-of-plane effects in this design.

Use 200 mm hollow concrete blocks of 15 MPa unit strength and Type S mortar. Grade 400 steel reinforcing bars (yield strength f_y = 400 MPa) and cold-drawn galvanized wire (ASWG) joint reinforcement are used for this design.



Wall dimensions:
l_w =8000 mm length
h_w = 6600 mm height
t = 190 mm thickness

SOLUTION:

1. Material properties

Steel (both reinforcing bars and joint reinforcement):
ϕ_s = 0.85 f_y = 400 MPa
Masonry:
ϕ_m = 0.6
S304.1 Table 4, 15 MPa concrete blocks and Type S mortar:
f'_m = 7.5 MPa (assume solid grouted masonry)

2. Load analysis

The wall needs to be designed for the following load effects:

- P_f = 230 kN axial load
- V_f = 630 kN seismic shear force
- $M_f = V_f * h$ = 630*6.6 ≈ 4160 kNm overturning moment at the base of the wall

Note that, according to NBCC 2005 Table 4.1.3.2, load combination for the dead load and seismic effects is 1.0*D + 1.0*E.

3. Minimum CSA S304.1 seismic reinforcement (see Section 2.5.4.7 and Table 2-2)
Since $I_E F_a S_a(0.2)$= 0.66 > 0.35, minimum seismic reinforcement is required (S304.1 Cl.10.15.2.2). See Example 4a for a detailed calculation of the S304.1 minimum seismic reinforcement.

4. Design for the combined axial load and flexure
A design for the combined effects of axial load and flexure will be performed using two different procedures: i) by considering uniformly distributed vertical reinforcement, and ii) by considering concentrated and distributed reinforcement.

<u>Distributed wall reinforcement (see Section C.1.1.2)</u>
This procedure assumes uniformly distributed vertical reinforcement over the wall length. The total vertical reinforcement area can be estimated, and the estimate can be revised until the moment resistance value is sufficiently large. After a few trial estimates, the total area of vertical reinforcement was determined as

A_{vt} = 3200 mm^2 > 1016 mm^2 (minimum seismic reinforcement) - OK

Try 16-15M bars for vertical reinforcement.
The wall is subjected to axial load

P_f = 230 kN

The approximate moment resistance for the wall section is given by:

$$\alpha_1 = 0.85 \qquad \beta_1 = 0.8$$

$$\omega = \frac{\phi_s f_y A_{vt}}{\phi_m f'_m l_w t} = \frac{0.85*400*3200}{0.6*7.5*8000*190} = 0.159$$

$$\alpha = \frac{P_f}{\phi_m f'_m l_w t} = \frac{230*10^3}{0.6*7.5*8000*190} = 0.034$$

$$c = \frac{\omega+\alpha}{2\omega+\alpha_1\beta_1} l_w = \frac{0.159+0.034}{2*0.159+0.85*0.8}(8000) = 1547 \text{ mm}$$

$$M_r = 0.5\phi_s f_y A_{vt} l_w\left(1+\frac{P_f}{\phi_s f_y A_{vt}}\right)\left(1-\frac{c}{l_w}\right) = 0.5*0.85*\frac{400}{1000}*3200*\frac{8000}{1000}\left(1+\frac{230*10^3}{0.85*400*3200}\right)\left(1-\frac{1544}{8000}\right)$$

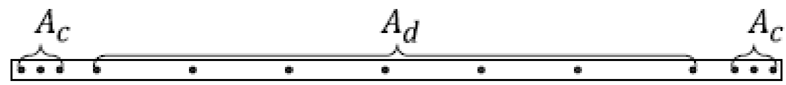
$M_r = 4253$ kNm > $M_f = 4160$ kNm OK

<u>Distributed and concentrated wall reinforcement (see Section C.1.1.1)</u>

This procedure assumes the same total reinforcement area, but the concentrated reinforcement is provided at the wall ends, and the remaining reinforcement is distributed over the wall length.

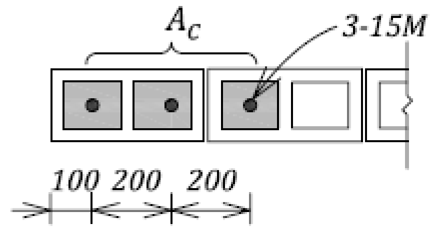
A_{vt} = 3200 mm^2

Concentrated reinforcement area at each wall end (3-15M bars in total, 1-15M in last 3 cells):
A_c = 600 mm^2



Distributed reinforcement
A_d = 3200-2*600=2000 mm^2

Distance from the wall end to the centroid of concentrated reinforcement
$d' = 300$ mm



The compression zone depth a:

$$a = \frac{P_f + \phi_s f_y A_d}{0.85\phi_m f'_m t} = \frac{230*10^3 + 0.85*400*2000}{0.85*0.6*7.5*190}$$ = 1252 mm

The masonry compression resultant C_r:

$C_m = (0.85\phi_m f'_m)(t \cdot a) = (0.85*0.6*7.5)(190*1252) = 910$ kN

The factored moment resistance M_r will be determined by summing up the moments around the centroid of the wall section as follows (see equation (3) in Section C.1.1.1)

$$M_r = \left[C_m(l_w - a)/2 + 2(\phi_s f_y A_c)(l_w/2 - d')\right]*10^{-6}$$

$$= \left[910*10^3*(8000-1252)/2 + 2*(0.85*400*600)(8000/2-300)\right]*10^{-6} \quad M_r = 4580 \text{ kNm}$$

The second procedure was used as a reference (to confirm the results of the first procedure). Both procedure give similar M_r values (4253 kNm and 4580 kNm by the first and second procedure respectively).

5. Find the diagonal tension shear resistance (see Section 2.3.2 and CSA S304.1 Cl.10.10.1).

Masonry shear resistance (V_m):

$b_w = 190$ mm overall wall thickness

$d_v \approx 0.8 l_w = 6400$ mm effective wall depth

$\gamma_g = 1.0$ solid grouted wall

$P_d = 0.9 P_f$ = 207 kN

$$v_m = 0.16\left(2 - \frac{M_f}{V_f d_v}\right)\sqrt{f'_m}$$ = 0.44 MPa

$$\frac{M_f}{V_f d_v} = \frac{4160}{630*6.4}$$ = 1.03 ≈ 1.0

$V_m = \phi_m (v_m b_w d_v + 0.25 P_d)\gamma_g$ = 0.6(0.44*190*6400+0.25*207*10^3)*1.0 = 352 kN

Steel shear resistance V_s (2-15M bond beam reinforcement at 1200 mm spacing):

$$V_s = 0.6\phi_s A_v f_y \frac{d_v}{s} = 0.6 * 0.85 * \frac{400}{1000} * 400 * \frac{6400}{1200}$$ = 435 kN

Total shear resistance

$V_r = V_m + V_s = 352 + 435 = 787$ kN

Since

$V_r = 787$ kN > $V_f = 630$ kN OK

This is a squat shear wall because $\frac{h_w}{l_w} = \frac{6600}{8000} = 0.825 \leq 1.0$. Maximum shear allowed on the section is (S304.1 Cl.10.10.1.3)

$$\max V_r = 0.4\phi_m \sqrt{f'_m} b_w d_v \gamma_g (2 - \frac{h_w}{l_w}) = 939 \text{ kN}$$

Since

$V_r < \max V_r$ OK

Note that a solid grouted wall is required, that is, $\gamma_g = 1.0$. A partially grouted wall would have $\gamma_g = 0.5$, so its shear capacity would not be adequate for this design.

6. Sliding shear resistance (see Section 2.3.3)

The factored in-plane sliding shear resistance V_r is determined as follows.

μ = 1.0 for a masonry-to-masonry or masonry-to-roughened concrete sliding plane

A_s = 3200 mm^2 total area of vertical wall reinforcement

$T_y = \phi_s A_s f_y$ = 0.85*3200*400 = 1088 kN

P_d = 207 kN

$P_2 = P_d + T_y$ = 207+1088 = 1295 kN

$V_r = \phi_m \mu P_2$ = 0.6*1.0*1295=777 kN

$V_r = 777$ kN > $V_f = 630$ kN OK

7. Capacity design check (see Section 2.5.2)

At this point, both the moment resistance M_r and the diagonal shear resistance V_r for the wall section have been determined. It is a good seismic design practice to design structural members so that a ductile flexural failure takes place before a shear failure has been initiated, that is, to follow the capacity design approach discussed in Section 2.5.2. <u>Note that CSA S304.1-04 does not formally require that capacity design approach be applied to reinforced masonry walls of conventional construction – it is mandatory only for “ductile walls”.</u>

In this case, the factored moment resistance is equal to

$M_r = 4253$ kNm

The nominal moment resistance can be estimated as follows

$$M_n = \frac{M_r}{\phi_s} = \frac{4253}{0.85} = 5004 \text{ kNm}$$

The shear force at the top of the wall that would cause an overturning moment equal to M_n is

$$V_{nb} = \frac{M_n}{h_w} = \frac{5004}{6.6} = 758 \text{ kN}$$

To ensure that a flexural failure takes place before the diagonal shear failure, it is required that (see Figure 2-22)

$V_{nb} \le V_r$

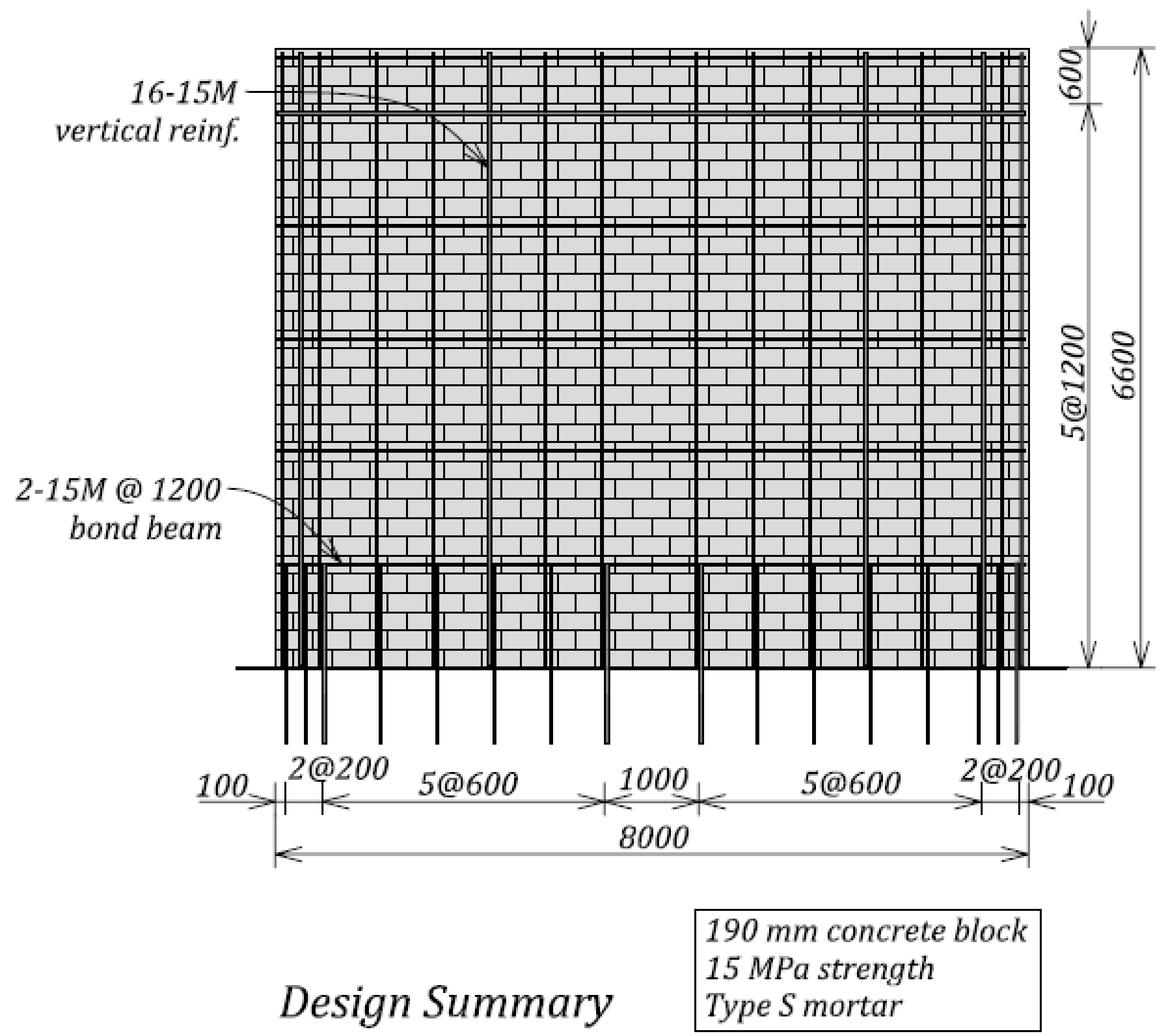
Since

$V_{nb} = 758 < V_r = 787$ kN

the capacity design criterion is satisfied (see discussion in Example 4a).

8. Design summary

The reinforcement arrangement for the wall under consideration is shown in the figure below. Note that the wall is solidly grouted. A bond beam (transfer beam) is provided atop the wall to ensure uniform shear transfer along the entire length (see Section 2.3.2.2).



10. Discussion

It is important to consider all possible behaviour modes and identify the one that governs in this design. There are three shear forces:

a) $V_{nb} = 758$ kN shear force corresponding to flexural failure

b) $V_r = 787$ kN diagonal tension shear resistance

c) $V_r = 777$ kN sliding shear resistance

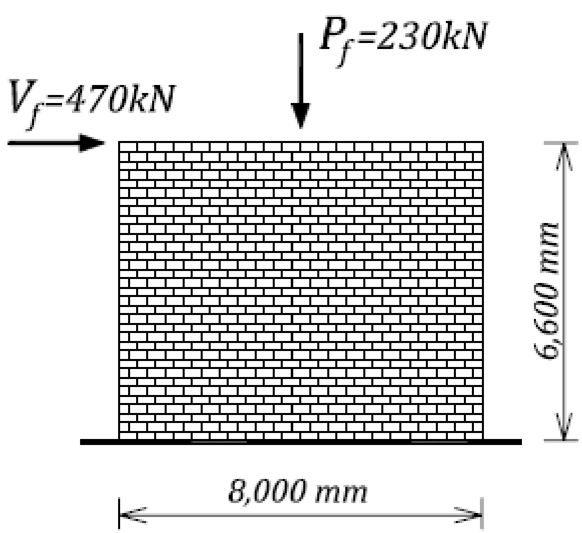
Since the shear force corresponding to flexural resistance is smallest of the three values, it can be concluded that the flexural failure mechanism is critical in this case, which is desirable for seismic design.

Note that CSA S304.1-04 Cl.10.2.8 prescribes the use of a reduced effective depth d for the flexural design of squat shear walls. This example deals with seismic design, and the wall reinforcement is expected to yield in tension, this provision was not followed since it would lead to a non-conservative design; instead, the actual effective depth was used for flexural design.

EXAMPLE 4c: Seismic design of a squat shear wall of moderate ductility

Design a single-storey squat concrete block shear wall shown on the figure below according to NBCC 2005 and CSA S304.1 seismic requirements for moderately ductile squat shear walls (note that the same shear wall was designed in Example 4b as a conventional construction). The building site is located in Ottawa, ON and the seismic hazard index $I_E F_a S_a(0.2)$ is 0.66. The wall is subjected to the total dead load of 230 kN (including the wall self-weight) and the in-plane seismic force of 470 kN; this reflects the higher R_d value of 2.0 that can be used for walls with moderate ductility. Consider the wall to be solid grouted. Neglect the out-of-plane effects in this design.

Use 200 mm hollow concrete blocks of 15 MPa unit strength and Type S mortar. Grade 400 steel reinforcing bars (yield strength f_y = 400 MPa) and cold-drawn galvanized wire (ASWG) joint reinforcement are used for this design.



Wall dimensions:
l_w =8000 mm length
h_w = 6600 mm height
t = 190 mm thickness

Note that the h/t ratio exceeds the S304.1 limit of 20 for moderately ductile squat shear walls (Cl.10.16.6.3).

SOLUTION:
Since

$$\frac{h_w}{l_w} = \frac{6600}{8000} = 0.825 \leq 1.0$$

this is a squat shear wall (S304.1 Cl.4.6.6). The wall is to be designed as a moderately ductile squat shear wall, and NBCC 2005 Table 4.1.8.9 specifies the following R_d and R_o values (see Table 1-13):

R_d = 2.0 and R_o = 1.5

The seismic shear force of 470 kN for a wall with moderate ductility ($R_d = 2.0$) was obtained by prorating the force of 630 kN from Example 4b which corresponded to a shear wall with conventional construction ($R_d = 1.5$), as follows

$$V_f = 630 * \frac{1.5}{2.0} \approx 470 \text{ kN}$$

1. Material properties
Steel (both reinforcing bars and joint reinforcement):
ϕ_s = 0.85 f_y = 400 MPa

Masonry:
ϕ_m = 0.6
From S304.1 Table 4, 15 MPa concrete blocks and Type S mortar:
f'_m = 7.5 MPa (assume solid grouted masonry)

2. Load analysis
The wall needs to be designed for the following load effects:
- P_f = 230 kN axial load
- V_f = 470 kN seismic shear force
- $M_f = V_f * h$ = 470*6.6 ≈ 3100 kNm overturning moment at the base of the wall

Note that, according to NBCC 2005 Table 4.1.3.2, the load combination for the dead load and seismic effects is 1.0*D + 1.0*E.

3. Minimum CSA S304.1 seismic reinforcement (see Section 2.5.4.7 and Table 2-2)
Since $I_E F_a S_a(0.2)$ = 0.66 > 0.35, minimum seismic reinforcement is required (Cl.10.15.2.2). See Example 4a for a detailed calculation of the S304.1 minimum seismic reinforcement.

4. Design for the combined axial load and flexure (see Section C.1.1.2).
A design for the combined effects of axial load and flexure will be performed by assuming uniformly distributed vertical reinforcement over the wall length. After a few trial estimates, the total area of vertical reinforcement was determined as
A_{vt} = 2200 mm^2 > 1016 mm^2 (minimum seismic reinforcement) - OK
and so 11-15M reinforcing bars can be used for vertical reinforcement in this design (total area of 2200 mm^2).

The wall is subjected to axial load P_f = 230 kN. Note that the load factor for the load combination with earthquake load is equal to 1.0.

The moment resistance for the wall section can be determined from the following equations (see Example 4b):
$\alpha_1 = 0.85 \quad \beta_1 = 0.8 \quad \omega = 0.109 \quad \alpha = 0.034 \quad c = 1273$ mm

$$M_r = 0.5\phi_s f_y A_{vt} l_w \left(1 + \frac{P_f}{\phi_s f_y A_{vt}}\right)\left(1 - \frac{c}{l_w}\right) = 0.5*0.85*\frac{400}{1000}*2200*\frac{8000}{1000}\left(1 + \frac{230*10^3}{0.85*400*2200}\right)\left(1 - \frac{1273}{8000}\right)$$

$M_r \cong 3290$ kNm > $M_f = 3100$ kNm OK

5. Height/thickness ratio check (see Section 2.5.4.4)
CSA S304.1-04 prescribes the following height-to-thickness (h/t) limit for the compression zone in moderately ductile squat shear walls (Cl.10.16.6.3):
$h/(t+10) < 20$, unless it can be shown for lightly loaded walls that a more slender wall is satisfactory for out-of-plane stability.

For this example,
h = 6600 mm (unsupported wall height)

t = 190 mm actual wall thickness
So,
$h/(t+10) = 6600/(190+10) = 33 > 20$
The height-to-thickness ratio for this wall exceeds the CSA S304.1 limits by a significant margin. However, CSA S304.1 permits the height-to-thickness restrictions for moderately ductile squat shear walls to be relaxed, provided that the designer can show that the out-of-plane wall stability is satisfactory.

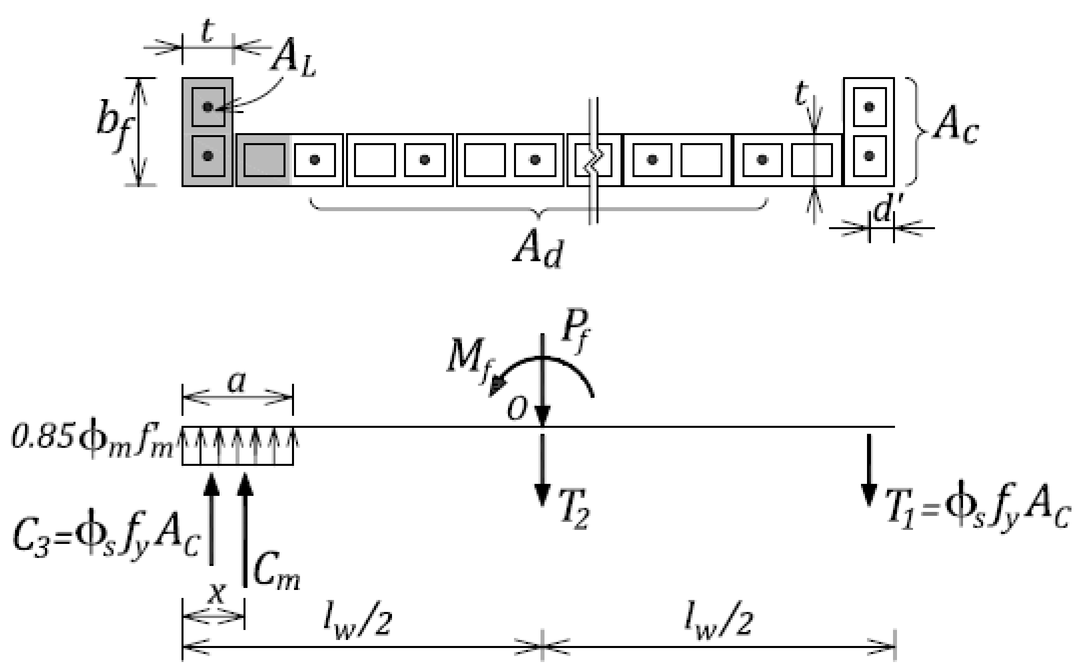
This is a lightly loaded wall in a single-storey building. The total dead load is 230 kN, which corresponds to the compressive stress of

$$f_c = \frac{P_f}{l_w t} = \frac{230*10^3}{8000*190} = 0.15 \text{ MPa}$$

This stress corresponds to only 2% of the masonry compressive strength f'_m which is equal to 7.5 MPa. In general, a compressive stress below 0.1 f'_m (equal to 0.75 MPa in this case) is considered to be very low.

The recommendations included in the commentary to Section 2.5.4.4 will be followed here. A possible solution involves the provision of flanges at the wall ends. The out-of-plane stability of the compression zone must be confirmed for this case.

Try an effective flange width $b_f = 390$ mm. The wall section and the internal force distribution is shown on the figure below.



This procedure assumes the same total reinforcement area A_{vt} as determined in step 4, but the concentrated reinforcement is provided at the wall ends, while the remaining reinforcement is distributed over the wall length.

A_{vt} = 2200 mm^2

Concentrated reinforcement area (2-15M bars at each wall end):

A_c = 400 mm^2

Distributed reinforcement area:

A_d =2200-2*400=1400 mm^2

Distance from the wall end to the centroid of concentrated reinforcement A_c:

$d' = 100$ mm

- Check the buckling resistance of the compression zone.

The area of the compression zone A_L:

$$A_L = \frac{P_f + \phi_s f_y A_d}{0.85\phi_m f'_m} = \frac{230*10^3 + 0.85*400*1400}{0.85*0.6*7.5} = 1.846*10^5 \text{ mm}^2$$

The depth of the compression zone a:

$$a = \frac{A_L - b_f * t + t^2}{t} = \frac{1.846*10^5 - (390*190) + 190^2}{190} = 772 \text{ mm}$$

The neutral axis depth:

$$c = \frac{a}{0.8} = 965 \text{ mm}$$

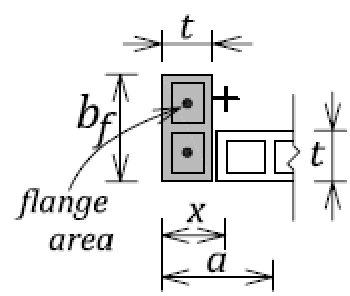
The centroid of the masonry compression zone:

$$x = \frac{t*\left(a^2/2\right) + (b_f - t)\left(t^2/2\right)}{A_L} = 326 \text{ mm}$$

In this case, the compression zone is L-shaped, however only the flange area will be considered for the buckling resistance check (see the shaded area shown on the figure below). This is a conservative approximation and it is considered to be appropriate for this purpose, since the gross moment of inertia is used.

Gross moment of inertia for the flange only:

$$I_{xg} = \frac{t*b_f^{\ 3}}{12} = \frac{190*390^3}{12} = 9.39*10^8 \text{ mm}^4$$



The buckling strength for the compression zone will be determined according to S304.1 Cl. 10.7.4.3, as follows:

$$P_{cr} = \frac{\pi^2 \phi_{er} E_m I}{(1+0.5\beta_d)(kH)^2} = 1017 \text{ kN}$$

where

$\phi_{er} = 0.75$

$k = 1.0$ pin-pin support conditions

$\beta_d = 0$ assume 100% seismic live load

$H = 6600$ mm wall height

$E_m = 850 f'_m = 6375$ MPa modulus of elasticity for masonry

- Find the resultant compression force (including the concrete and steel component).

$P_{fb} = C_m + \phi_s f_y A_c = 706*10^3 + 0.85*400*400 = 842$ kN

where

$C_m = (0.85\phi_m f'_m)A_L = (0.85*0.6*7.5)(1.846*10^5) = 706$ kN

- Confirm that the out-of-plane buckling resistance is adequate.

Since

$P_{fb} = 842$ kN < $P_{cr} = 1017$ kN

it can be concluded that the out-of-plane buckling resistance is adequate and so the flanged section can be used for this design. This is in compliance with S304.1 Cl.10.16.6.3, despite the fact that the h/t ratio for this wall is 33, which exceeds the CSA S304.1-prescribed limit of 20.

4a. Design the flanged section for the combined axial load and flexure – consider distributed and concentrated wall reinforcement (see Section C.1.1.1).

The key design parameters for this calculation were determined in step 5 above. The factored moment resistance M_r will be determined by summing up the moments around the centroid of the wall section as follows

$$M_r = C_m(l_w/2 - x) + 2(\phi_s f_y A_c)(l_w/2 - d') = 706*10^3*(8000/2-326)+2*(0.85*400*400)*(8000/2-100)$$

$M_r = 3655*10^6\, Nmm = 3655$ kNm

Since

$M_r = 3655$ kNm > $M_f = 3100$ kNm OK

6. The diagonal tension shear resistance (see Section 2.3.2 and CSA S304.1 Cl.10.10.1)

Masonry shear resistance (V_m):

$b_w = 190$ mm overall wall thickness

$d_v \approx 0.8 l_w = 6400$ mm effective wall depth

$\gamma_g = 1.0$ solid grouted wall

$P_d = 0.9 P_f$ = 207 kN

$v_m = 0.16(2 - \frac{M_f}{V_f d_v})\sqrt{f'_m}$ = 0.44 MPa

$\frac{M_f}{V_f d_v} = \frac{3100}{470*6.4}$ = 1.03 ≈ 1.0

$V_m = \phi_m(v_m b_w d_v + 0.25 P_d)\gamma_g$ = 0.6(0.44*190*6400+0.25*207*10^3)*1.0 = 352 kN

Steel shear resistance V_s:

Assume 2-15M bond beam reinforcement at 1200 mm spacing, so

$A_v = 400$ mm^2

$s = 1200$ mm

Horizontal reinforcement area per metre:

$A_h' = \frac{A_v}{s}*1000 = \frac{400}{1200}*1000 = 333$ mm^2/m

$V_s = 0.6\phi_s A_v f_y \frac{d_v}{s} = 0.6*0.85*\frac{400}{1000}*400*\frac{6400}{1200}$ = 435 kN

Total diagonal shear resistance

$V_r = V_m + V_s = 352 + 435 = 787$ kN

$V_r = 787$ kN > $V_f = 470$ kN OK

Maximum shear allowed on the section is (S304.1 Cl.10.10.1.3)

$$\max V_r = 0.4\phi_m \sqrt{f'_m} b_w d_v \gamma_g (2 - \frac{h_w}{l_w}) = 939 \text{ kN}$$

Since

$V_r < \max V_r$ OK

Note that CSA S304.1 Cl.10.16.6.2 requires that the method by which the shear force is applied to the wall shall be capable of applying shear force uniformly over the wall length. This can be achieved by providing a continuous bond beam at the top of the wall, as discussed in Section 2.3.2.2 (see Figure 2-16).

7. Sliding shear resistance (see Section 2.3.3)

The factored in-plane sliding shear resistance V_r is determined as follows.

μ = 1.0 for a masonry-to-masonry or masonry-to-roughened concrete sliding plane

A_s = 2200 mm^2 total area of vertical wall reinforcement

$T_y = \phi_s A_s f_y$ = 0.85*2200*400 = 748 kN

P_d = 207 kN

$P_2 = P_d + T_y$ = 207+748 = 955 kN

$V_r = \phi_m \mu P_2$ = 0.6*1.0*955 = 573 kN

$V_r = 573$ kN > $V_f = 470$ kN OK

Note that $V_r = 573$ kN < $V_{nb} = 787$ kN (this indicates that the sliding shear resistance governs over the diagonal tension shear resistance).

8. Reinforcement requirements for moderately ductile squat shear walls (see Section 2.5.4.8)

CSA S304.1-04 Cl.10.16.6.6 introduced the following new requirements for the amount of reinforcement in moderately ductile squat shear walls:

<u>Vertical reinforcement ratio ρ_v</u>

Actual vertical reinforcement ratio ρ_{vflex} based on the flexural design requirements (see step 4):

$$\rho_{vflex} = \frac{A_{vt}}{l_w * t} = \frac{2200}{8000*190} = 1.447*10^{-3}$$

Minimum $\rho_{v\min}$ value set by CSA S304.1 Cl.10.16.6.6.1:

$$\rho_{v\min} \geq \frac{V_f - P_f}{\phi_s \cdot b_w \cdot l_w \cdot f_y} = \frac{470*10^3 - 230*10^3}{0.85*190*8000*400} = 0.464*10^{-3}$$

Since

ρ_{vflex} = 1.447*10^{-3} > $\rho_{v\min}$ = 0.464*10^{-3}

Therefore, the amount of vertical reinforcement determined based on the flexural design requirements (11-15M) is OK.

Horizontal reinforcement ratio ρ_h

ρ_h should be greater of

a) the minimum value set by CSA S304.1 Cl.10.16.6.6.2:

$$\phi_s \rho_{h\min} \geq \phi_s \rho_v + \frac{P_f}{b_w l_w f_y} = 0.85 * 4.644 * 10^{-4} + \frac{230 * 10^3}{190 * 8000 * 400} = 0.773 * 10^{-3}$$

(note that the vertical reinforcement ratio used in this relation is the one that governed above, that is, $\rho_v = \rho_{v\min}$ = 4.644*10^-4^)

and

b) the value determined in accordance with Cl.10.10 based on the shear resistance requirements

$$\phi_s \rho_{hshear} = \frac{\phi_s A_h}{b_w * h_w} = \frac{0.85 * 2131}{190 * 6600} = 1.44 * 10^{-3}$$

where A_h is the total area of horizontal reinforcement along the wall height, that is,

$A_h = A_h' * d_v$ = 333*6.4=2131 mm^2

$A_h' = 333$ mm^2/m (see step 6)

In this case,

$\phi_s \rho_{h\min}$ =0.773*10^{-3} < $\phi_s \rho_{hshear}$ =1.44*10^{-3}

This indicates that the CSA S304.1 shear resistance requirement governs. The amount of horizontal reinforcement (2-15M bond beam reinforcement bar at 1200 mm spacing) is adequate.

9. Capacity design check (see Section 2.5.2)

At this point, both the moment resistance M_r and the diagonal shear resistance V_r for the wall section have been determined. S304.1 Cl.10.16.3.3 requires that ductile reinforced masonry shear walls be designed so that flexural failure takes place before shear failure has been initiated, that is, to follow the capacity design approach (see Section 2.5.2 for more details).

In this case, the factored moment resistance is equal to

$M_r = 3655$ kNm

The nominal moment resistance can be estimated as follows

$$M_n = \frac{M_r}{\phi_s} = \frac{3655}{0.85} = 4300 \text{ kNm}$$

The shear force at the top of the wall that would cause an overturning moment equal to M_n is

$$V_{nb} = \frac{M_n}{h_w} = \frac{4300}{6.6} = 652 \text{ kN}$$

In order to ensure that flexural failure takes place before the diagonal shear failure, it is required that (see Figure 2-22)

$V_{nb} \leq V_r$

Since

$V_{nb} = 652 < V_r = 787$ kN

the capacity design criterion is satisfied (see discussion in Example 4a).

10. Shear resistance at the web-to-flange interface (see Section C.2 and Cl.7.11.4).

The factored shear stress at the web-to-flange interface is equal to the larger of horizontal and vertical shear stress, as shown below.

Horizontal shear:

$$v_f = \frac{V_f}{t_e l_w} = \frac{470*10^3}{190*8000} = 0.31 \text{ MPa}$$

where t_e = 190 mm (effective wall thickness)

Vertical shear (caused by the resultant compression force P_{fb} calculated in Step 5):

$$v_f = \frac{P_{fb}}{b_w * h_w} = \frac{842*10^3}{190*6600} = 0.67 \text{ MPa} \qquad \text{governs}$$

Masonry shear resistance:

$v_m = 0.44$ MPa (see step 6)

Since

$v_f = 0.67$ MPa > $\phi_m v_m = 0.26$ MPa

shear reinforcement at the web-to-flange interface is required. Since the horizontal reinforcement consists of 2-15M bars @ 1200 mm spacing, both bars can be extended into the flange (90° hook), and so

$$v_s = \frac{\phi_s A_b f_y}{s \cdot t_e} = \frac{0.85*2*200*400}{1200*190} = 0.60 \text{ MPa}$$

The total shear resistance

$v_r = \phi_m v_m + v_s = 0.26 + 0.60 = 0.86$ MPa

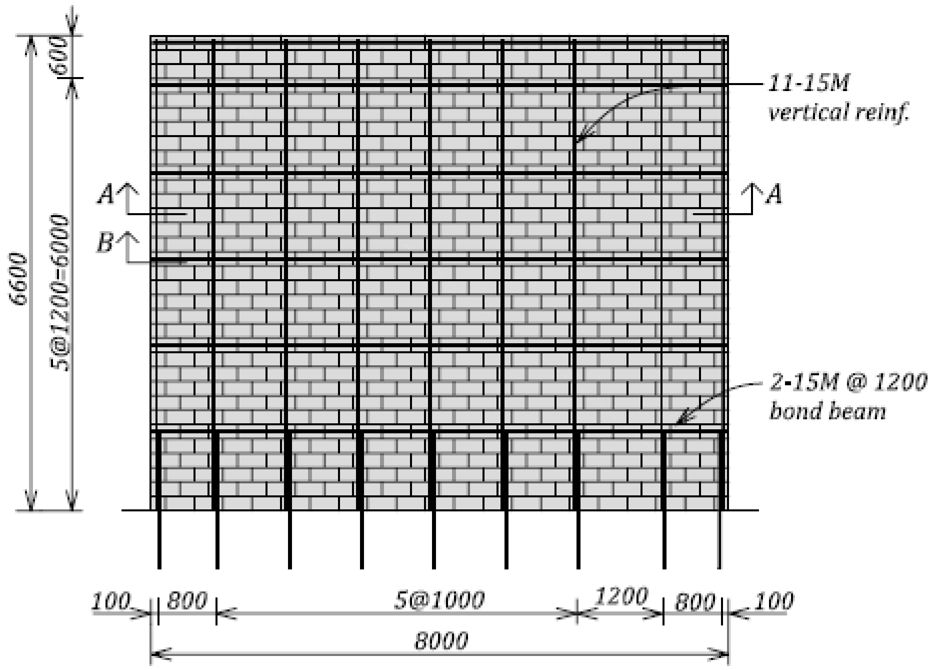
Since

$v_f = 0.67$ MPa < $v_r = 0.86$ MPa

the shear resistance at the web-to-flange interface is satisfactory.

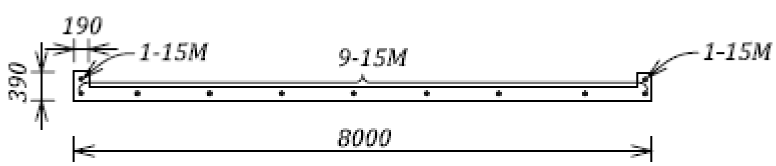
12. Design summary

The reinforcement arrangement for the wall under consideration is shown in the figure below. Note that the wall is solid grouted.

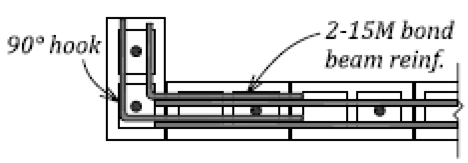


Design Summary

190 mm concrete block
15 MPa strength
Type S mortar



Section A-A



Section B-B

13. Discussion

It is important to consider all possible behaviour modes and identify the one that governs in this design. There are three shear forces:

a) $V_{nb} = 652$ kN shear force corresponding to flexural failure

b) $V_r = 787$ kN diagonal tension shear resistance

c) $V_r = 573$ kN sliding shear resistance

Since the sliding shear resistance value is smallest, it can be concluded that the sliding shear mechanism is critical in this case, which is common for seismic design of squat shear walls. Sliding shear resistance can be increased by roughening the wall-to-foundation interface (in which case the frictional coefficient can be increased to μ = 1.4) or by providing shear keys. Alternatively, additional dowels could be provided at the base of the wall, however this would result in an increase in the moment resistance. The designer would need to ensure that the capacity design criterion discussed in step 9 is satisfied.

Note that CSA S304.1-04 Cl.10.2.8 prescribes the use of reduced effective depth d for flexural design of squat shear walls. Since this example deals with seismic design and essentially all the wall reinforcement is expected to yield in tension, this provision was not used as it is expected to result in additional vertical reinforcement, which would increase the moment capacity and possibly lead to a more brittle diagonal shear failure.

Note that the S304.1 ductility check is not prescribed for moderately ductile squat shear walls (this requirement applies to the narrower flexural shear walls of moderate ductility per Cl.10.16.5.2.3).

This example shows that an addition of flanges can be effective in preventing the out-of-plane buckling of moderately ductile squat shear walls. This is in compliance with S304.1 Cl.10.16.6.3, despite the fact that the h/t ratio for this wall is 33, which exceeds the CSA S304.1-prescribed limit of 20.

The last two examples provide an opportunity for comparing the total amount of vertical reinforcement required for a squat shear wall of conventional construction (Example 4b) and a moderately ductile squat shear wall (this example). It is noted that the moderately ductile wall has less vertical reinforcement (11-15M bars) than a similar wall of conventional construction (16-15M bars); this reduction amounts to approximately 30%.

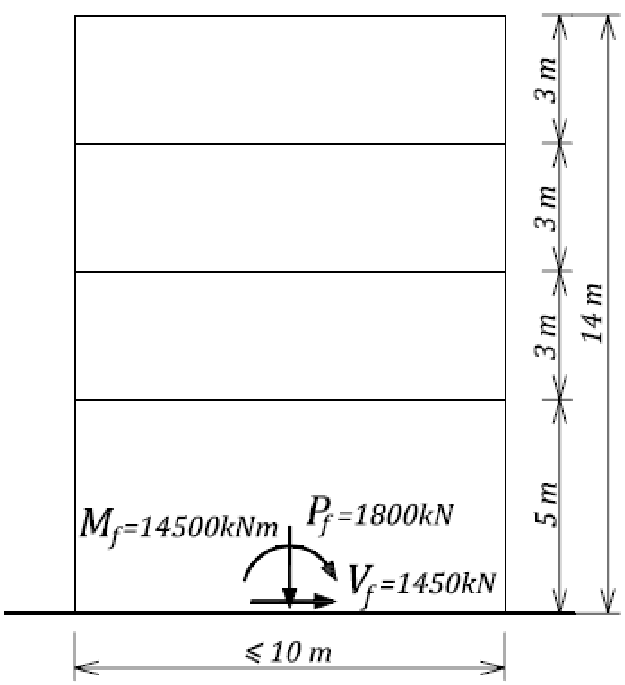
<u>EXAMPLE 5a</u>: Seismic design of a flexural shear wall of $\boxed{\text{limited ductility}}$

Perform the seismic design of a shear wall X_1, which is a part of the building discussed in Example 2. The wall is four storeys high, with the total height of 14 m, and due to its height must be designed either as a "limited ductility" or a "moderate ductility" shear wall per NBCC 2005 Table 4.1.8.9 (same as Table 1-13 in Chapter 1 of this document).

The section at the base of the wall is subjected to the total dead load of 1800 kN, the in-plane seismic shear force of 1450 kN, and the overturning moment of 14500 kNm. Select the wall dimensions (length and thickness) and the reinforcement such that the CSA S304.1 Cl.10.16.4 seismic design requirements for limited ductility shear walls are satisfied. Due to architectural constraints, the wall length should not exceed 10 m, and a rectangular (unflanged) wall section should be used.

Use hollow concrete blocks of 20 MPa unit strength and Type S mortar. Consider the wall as solid grouted. Grade 400 steel reinforcement (yield strength f_y = 400 MPa) is used for this design.

Note: the wall dead load was calculated based on the tributary area (3.4 m by 13.4 m) at each floor level (see a typical floor plan shown in Example 2), plus the wall self-weight.



<u>SOLUTION:</u>

1. Material properties

Steel (both reinforcing bars and joint reinforcement):

ϕ_s = 0.85 f_y = 400 MPa

Masonry:

ϕ_m = 0.6

S304.1 Table 4, 20 MPa concrete blocks and Type S mortar:

f'_m = 10.0 MPa (assume solid grouted masonry)

2. Load analysis

The section at the base of the wall needs to be designed for the following load effects:

- P_f = 1800 kN axial load
- V_f = 1450 kN seismic shear force
- M_f = 14500 kNm overturning moment

According to S304.1 Cl.4.6.4, this is a flexural shear wall because h_w = 14000 mm height and l_w = 10000 mm length, and

$$\frac{h_w}{l_w} \geq \frac{14000}{10000} \geq 1.4 > 1.0$$

and so the CSA S304.1 seismic design requirements for limited ductility (flexural) shear walls should be followed.

3. Determine the required wall thickness based on the S304.1 height-to-thickness requirements (Cl.10.16.4.1.2, see Section 2.5.4.4)

CSA S304.1-04 prescribes the following height-to-thickness (h/t) limit for the compression zone in limited ductility shear walls:

$h/(t+10) < 18$

For this example,

h = 5000 mm (the largest unsupported wall height)

So,

$t \geq h/18 - 10 = 268$ mm

Therefore, in this case the only possible wall thickness is

$t = 290$ mm

Alternatively, the designer may wish to consider a flanged wall section with smaller thickness. This is possible, except that (s)he would need to prove that the out-of-plane wall stability is not a concern (see Example 5b).

4. Determine the wall length based on the shear design requirements.

Designers may be requested to determine the wall dimensions (length and thickness) based on the design loads. In this case, the thickness is governed by the height-to-thickness ratio requirements, and the length can be determined from the maximum shear resistance for the wall section. The shear resistance for flexural walls cannot exceed the following limit (S304.1 Cl.10.10.1.1):

$V_r \leq \max V_r = 0.4\phi_m \sqrt{f'_m} b_w d_v \gamma_g$

$\gamma_g = 1.0$ solid grouted wall (required for plastic hinge zone)

$b_w = 290$ mm overall wall thickness

$d_v \approx 0.8 l_w$ effective wall depth

Set

$V_r = V_f = 1450$ kN

and so

$$l_w > \frac{V_f}{0.4\phi_m\sqrt{f'_m}b_w(0.8)\gamma_g} = \frac{1450*10^3}{0.4*0.6*\sqrt{10}*290*0.8*1.0} = 8235 \text{ mm}$$

Therefore, based on the shear design requirements the designer could select the wall length of 8.4 m. However, a preliminary capacity design check indicated that a minimum wall length of nearly 10 m was required, thus try

$l_w = 10000$ mm

which gives

$\max V_r = 1760$ kN

5. Minimum CSA S304.1 seismic reinforcement requirements (see Table 2-2)

Since $I_E F_a S_a(0.2)$ = 0.95 > 0.35, it is required to provide minimum seismic reinforcement (S304.1 Cl.10.15.2.2). See Example 4a for a detailed discussion on the S304.1 minimum seismic reinforcement requirements.

6. Design for the combined axial load and flexure (see Section C.1.1.2).

Design for the combined effects of axial load and flexure will be performed by assuming uniformly distributed vertical reinforcement over the wall length. After a few trial estimates, the total area of vertical reinforcement was determined as follows

A_{vt} = 6000 mm^2

20-20M reinforcing bars can be used for vertical reinforcement in this design, and the average spacing is equal to

$$s \le \frac{10000-200}{19} = 516 \text{ mm}$$

Since the amount of vertical reinforcement is significant, it is required to check the maximum reinforcement area per S304.1 Cl.10.15.3 (see Table 2-2).

Since $s = 516mm$ < $4t = 4*290 = 1160mm$

$A_{s\max} = 0.02A_g = 0.02(290*10^3) = 5800$ mm^2/m

This corresponds to the total reinforcement area of approximately 58000 mm^2 for a 10 m long wall; this is significantly larger than the estimated area of vertical reinforcement.

The wall is subjected to axial load P_f = 1800 kN. The moment resistance for the wall section can be determined from the following equations (see Section C.1.1.2):

$\alpha_1 = 0.85 \quad \beta_1 = 0.8 \quad \omega = 0.12 \quad \alpha = 0.1 \quad c \approx 2400$ mm

$$M_r = 0.5\phi_s f_y A_{vt} l_w\left(1+\frac{P_f}{\phi_s f_y A_{vt}}\right)\left(1-\frac{c}{l_w}\right) = 0.5*0.85*\frac{400}{1000}*6000*\frac{10000}{1000}\left(1+\frac{1800*10^3}{0.85*400*6000}\right)\left(1-\frac{2400}{10000}\right)$$

$M_r = 14600$ kNm > $M_f = 14500$ kNm OK

7. Perform the CSA S304.1 ductility check (see Section 2.5.4.3).

To satisfy the CSA S304.1 ductility requirements for limited ductility shear walls (Cl.10.16.4.1.4), neutral axis depth ratio (c/l_w) should be less than the following limit:

$c/l_w < 0.2$ when $h_w/l_w < 6$

In this case, the neutral axis depth

c = 2400 mm

and so

$c/l_w = 2400/10000 = 0.24 > 0.2$

Therefore, the CSA S304.1 ductility requirement is not satisfied. However, Cl.10.16.4.1.4 also states that the maximum compressive strain in masonry in the plastic hinge zone shall be shown to not exceed 0.0025 at the desired ductility level.

At this point, the designer can use one of the following two alternative approaches to check whether the ductility is adequate per CSA S304.1 Cl.10.16.4.1.4:

1) Find the required wall length such that the c/l_w limit prescribed in the CSA S304.1 ductility criteria is satisfied.

The wall length can be estimated from Table D-2, which provides c/l_w ratios for different input parameters (α and ω). By inspection, it can be concluded that $c/l_w < 0.2$ when $\alpha \le 0.1$. Let us try to estimate the wall length based on this criterion.

Since

$$\alpha = \frac{1667 * P_f}{f'_m l_w t}$$

set

$$\alpha = 0.09 < 0.1$$

and so

$$l_w = \frac{1667 * P_f}{f'_m * \alpha * t} = \frac{1667 * 1800}{10.0 * 0.09 * 290} = 11496 \text{ mm}$$

Therefore, we can select an increased wall length $l_w = 11600$ mm.

2) Calculate the masonry strain in the extreme compression fibre based on the given design loads, and prove that its value is less than 0.0025.

This check will be performed based on the procedure explained in Section B.2 (see Figure B-5). The maximum displacement in the wall X_1 at the roof level was determined in Example 2 (step 8), that is,

$$\Delta_{\max} = 46mm$$

Note that the above value includes only translational displacement component. Since the wall X_1 is located close to the centre of resistance, the torsional displacement component is not significant. In the case of wall X_1, torsional displacement has a different direction from the translational displacement and (if included) the total displacement would be less than the translational one.

The maximum displacement $\Delta_{\max}$ is equal to the sum of elastic displacement at the onset of steel yielding Δ_y and the plastic (post-yield) displacement Δ_p, that is,

$$\Delta_{\max} = \Delta_y + \Delta_p$$

The yield curvature (corresponding to the onset of yielding in steel reinforcement) can be estimated as follows

$\varphi_y = \dfrac{0.0035}{l_w} = \dfrac{0.0035}{10000} = 3.5 * 10^{-7}$ The elastic displacement at the effective height can be estimated as

$$\Delta_{ye} = \frac{\varphi_y h_e^{\,2}}{3} = \frac{(3.5 * 10^{-7})(10000)^2}{3} = 11.7mm$$

The elastic displacement at the top of the wall is equal to (see the discussion in Example 2, step 8)

$$\Delta_y = 1.5 * \Delta_{ye} = 1.5 * 11.7 = 17.5mm$$

So, the plastic displacement can be determined as

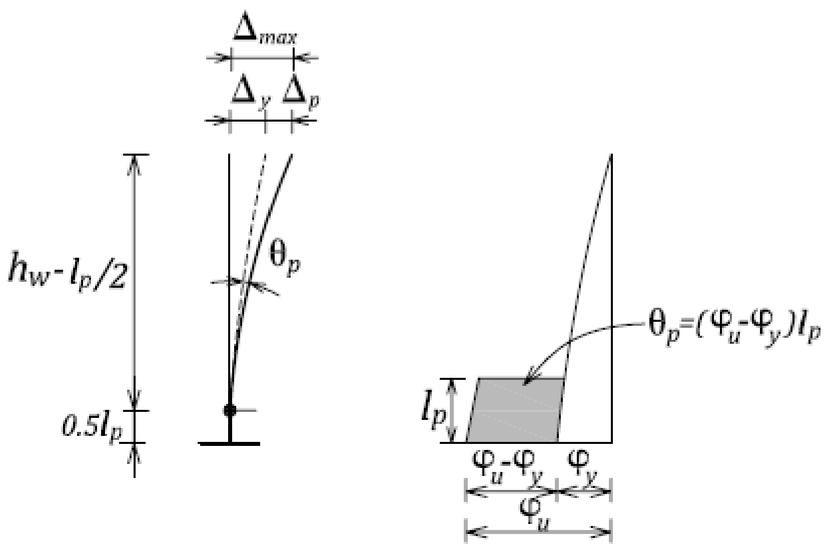
$$\Delta_p = \Delta_{\max} - \Delta_y = 46 - 17.5 = 28.5mm$$

The plastic rotation θ_p can be found from the plastic displacement at the top and assuming that the plastic hinge has developed at the base is equal to (see the figure below)

$$\theta_p = \frac{\Delta_p}{h_w - \frac{l_p}{2}} = \frac{28.5}{14000 - \frac{5000}{2}} = 2.48 * 10^{-3} \text{ rad}$$

where the plastic hinge length to be used for ductility calculations has been estimated as

$$l_p = 0.5 l_w = 0.5 * 10000 = 5000mm$$



The maximum curvature can be determined from the following relationship between the rotation and the curvature:

$$\theta_p = (\varphi_u - \varphi_y) * l_p$$

and so

$$\varphi_u - \varphi_y = \frac{\theta_p}{l_p} = \frac{2.48 * 10^{-3}}{5000} = 4.96 * 10^{-7}$$

The ultimate curvature can then be determined as

$$\varphi_u = 4.96 * 10^{-7} + 3.5 * 10^{-7} = 8.46 * 10^{-7}$$

The maximum compressive strain in masonry can be determined from the following equation

$$\varphi_u = \frac{\varepsilon_m}{c}$$

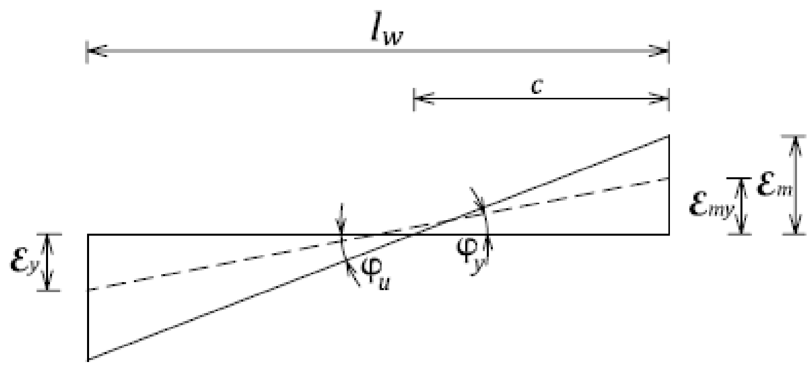
where

$c = 2400$ mm neutral axis depth (see step 6) and so

$$\varepsilon_m = \varphi_u * c = (8.46 * 10^{-7})(2400) \approx 0.002$$

It should be noted that this procedure uses an assumption that the neutral axis depth c has the same value at the onset of yielding (corresponding to strain ε_y) and at the ultimate (corresponding to strain ε_m); this is not true, however it does not significantly influence the accuracy of numerical results.

Since $\varepsilon_m = 0.002 < 0.0025$ it can be concluded that the wall satisfies the CSA S304.1 ductility requirements and that it is not necessary to increase its length. Therefore, the wall length $l_w = 10000$ mm will be used in the next steps. It should be noted that a larger wall length obtained from the first approach ($l_w = 11600$ mm) would have resulted in a reduced amount of vertical and horizontal reinforcement for the same flexural and shear design requirements, and would be a viable design solution had the wall length not been limited to 10 m due to architectural constraints.



8. The diagonal tension shear resistance and capacity design check (see Section 2.3.2 and CSA S304.1 Cl.10.10.1)

Masonry shear resistance (V_m):

$b_w = 290$ mm overall wall thickness

$d_v \approx 0.8 l_w = 8000$ mm effective wall depth

$\gamma_g = 1.0$ solid grouted wall

$P_d = 0.9 P_f$ = 1620 kN

$v_m = 0.16(2 - \frac{M_f}{V_f d_v})\sqrt{f'_m}$ = 0.51 MPa

Since

$\frac{M_f}{V_f d_v} = \frac{14500}{1450 * 8.0}$ = 1.25 > 1.0

use $\frac{M_f}{V_f d_v} = 1.0$

$V_m = \phi_m (v_m b_w d_v + 0.25 P_d)\gamma_g$ = 0.6(0.51*290*8000+0.25*1620*10^3)*1.0 = 953 kN

S304.1 Cl.10.16.3.3 requires that ductile reinforced masonry shear walls be designed according to the capacity design approach (see Section 2.5.2 for more details). According to that approach, the shear capacity should exceed the shear corresponding to the nominal moment resistance (see Figure 2-22), as follows

$M_n = \frac{M_r}{\phi_s} = \frac{14600}{0.85} = 17176$ kNm

where

$M_r = 14600$ kNm the factored moment resistance (see Step 6).

Shear force acts at the effective height h_e, that is, distance from the base of the wall to the resultant of all seismic forces acting at floor levels. h_e can be determined as follows

$$h_e = \frac{M_f}{V_f} = 10.0 \text{ m}$$

The shear force V_{nb} that would cause the overturning moment equal to M_n can be found as follows

$$V_{nb} = \frac{M_n}{h_e} = \frac{17176}{10.0} = 1718 \text{ kN}$$

This is less than the maximum shear allowed on the section (S304.1 Cl.10.10.1.1)

$$\max V_r = 0.4\phi_m \sqrt{f'_m} b_w d_v \gamma_g = 1760 \text{ kN} \quad \text{OK}$$

Thus the required steel shear resistance is

$$V_s = V_r - V_m = 1718 - 953 = 765 \text{ kN}$$

The required amount of reinforcement can be found from the following equation

$$\frac{A_v}{s} = \frac{V_s}{0.6\phi_s f_y d_v} = \frac{765 * 10^3}{0.6 * 0.85 * 400 * 8000} = 0.47$$

Try 2-15M bond beam reinforcing bars at 800 mm spacing ($A_v = 400$ mm^2 and $s = 800$ mm):

$$\frac{A_v}{s} = \frac{400}{800} = 0.5 > 0.47 \quad \text{OK}$$

Steel shear resistance V_s:

$$V_s = 0.6\phi_s A_v f_y \frac{d_v}{s} = 0.6 * 0.85 * \frac{400}{1000} * 400 * \frac{8000}{800} = 816 \text{ kN}$$

Total diagonal shear resistance:

$$V_r = V_m + V_s = 953 + 816 = 1769 \text{ kN}$$

Since

$$V_r = 1769 \text{ kN} > V_f = 1450 \text{ kN} \qquad \text{OK}$$

In conclusion, both the shear design requirements and the capacity design requirements have been satisfied.

9. Sliding shear resistance (see Section 2.3.3)

The factored in-plane sliding shear resistance V_r is determined as follows:

μ = 1.0 for a masonry-to-masonry or masonry-to-roughened concrete sliding plane

A_s = 6000 mm^2 total area of vertical wall reinforcement

$T_y = \phi_s A_s f_y$ = 0.85*6000*400 = 2040 kN

$P_d = 0.9 P_f$ = 1620 kN

$P_2 = P_d + T_y$ = 1620+2040 = 3660 kN

$V_r = \phi_m \mu P_2$ = 0.6*1.0*3660 = 2196 kN

$V_r = 2196$ kN > $V_f = 1450$ kN OK

Also,

$V_r = 2196$ kN > $V_{nb} = 1718$ kN (capacity design check)

10. CSA S304.1 seismic detailing requirements for limited ductility walls – plastic hinge region

According to Cl.10.16.4.1.1, the required height of the plastic hinge region for limited ductility shear walls (for which special detailing is required) must be greater than (see Table 2-4)

$l_p = l_w / 2 = 10.0/2 = 5.0$ m

or

$l_p = h_w / 6 = 14.0 / 6 = 2.3$ m

(note that h_w denotes the total wall height)

Thus,

$l_p = 5.0$ m governs

Reinforcement detailing requirements for the plastic hinge region of limited ductility shear walls are:

1. ***The wall in the plastic hinge region must be solid grouted (Cl.10.16.4.1.3, see Table 2-4).***

2. ***Horizontal reinforcement requirements (see Figure 2-31)***

a) Reinforcement spacing should not exceed the following limits (Cl.10.16.4.3.3), see Table 2-2:

$s \leq 1200$ mm or

$s \leq l_w/2 = 10000/2 = 5000$ m

Since the lesser value governs, the maximum permitted spacing is

$s \leq 1200$ mm

According to the design (see step 8), the horizontal reinforcement consists of 2-15M bars at 800 mm spacing - OK

b) Detailing requirements (Cl.10.16.4.3.3), see Table 2-3:

Horizontal reinforcement shall not be lapped within

600 mm or

c = 2400 mm (the neutral axis depth)

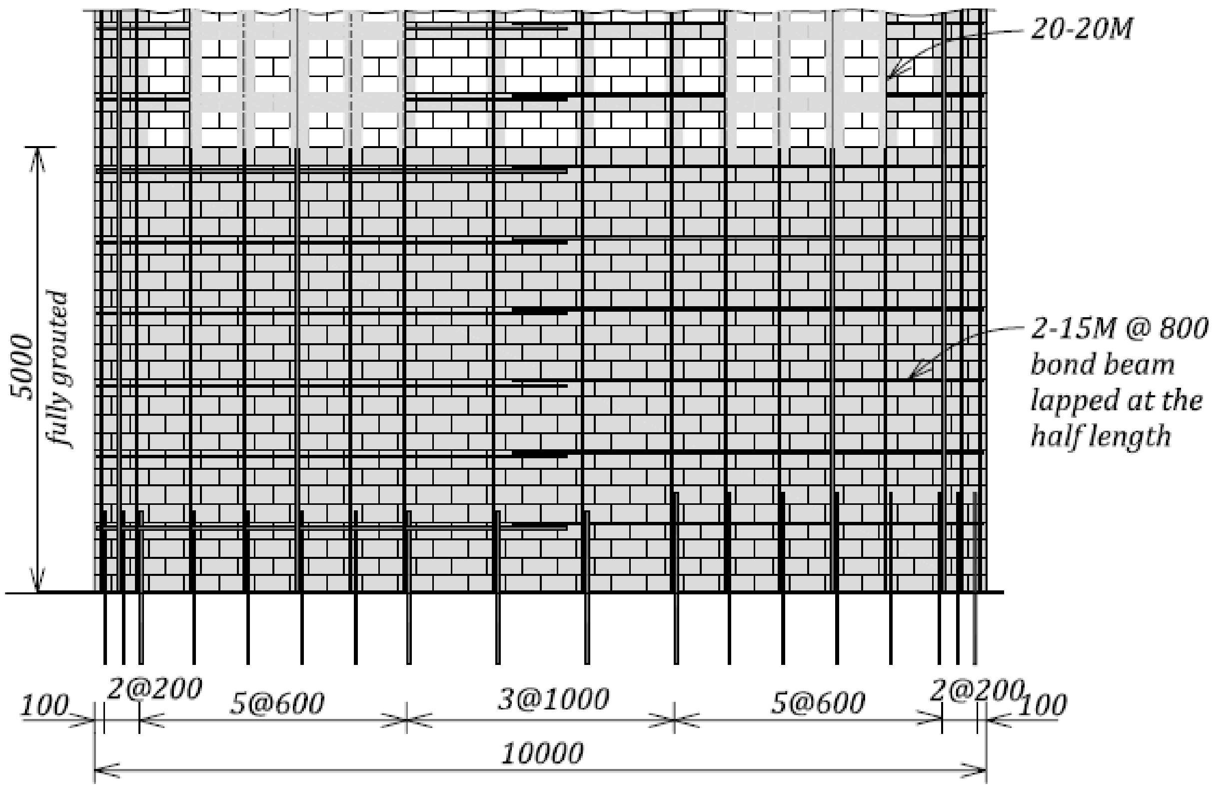
whichever is greater, from the end of the wall. In this case, the reinforcement should not be lapped within the distance c = 2400 mm from the end of the wall. The horizontal reinforcement can be lapped at the wall half-length.

3. Vertical reinforcement requirements (see Table 2-3).

There are no special detailing requirements for vertical reinforcement in limited ductility shear walls.

11. Design summary

Reinforcement arrangement for the wall under consideration is summarized on the figure below. Note that the shear wall of limited ductility must be solid grouted in plastic hinge region, but it may be partially grouted outside the plastic hinge region (this depends on the design forces).



12. Discussion

It is important to consider all possible behaviour modes and identify the one that governs in this design. The following three shear resistance values need to be considered:

a) $V_{nb} = 1718$ kN shear force corresponding to flexural failure

b) $V_r = 1769$ kN diagonal tension shear resistance

c) $V_r = 2196$ kN sliding shear resistance

Since the shear force corresponding to the flexural resistance is smallest of the three values, it can be concluded that the flexural failure mechanism is critical in this case, which is desirable for the seismic design.

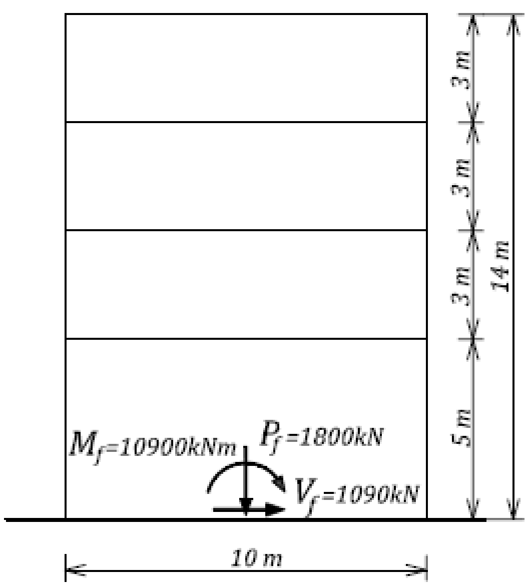
Had the design specified a shear wall of conventional construction, the same amount of vertical and horizontal reinforcement would have been required, but none of the special detailing discussed in step 10 would have been required. Also, the CSA S304.1 ductility check discussed in step 7 is not required for shear walls of conventional construction.

EXAMPLE 5b: Seismic design of a flexural shear wall of $\boxed{\text{moderate ductility}}$

Perform the seismic design of a shear wall X_1 located at the rear side of the building discussed in Example 2 and Example 5a. Try to use the same wall dimensions as in Example 5a, that is, 10 m length and 290 mm thickness.

The section at the base of the wall is subjected to the total dead load of 1800 kN (including the wall self-weight), the in-plane seismic shear force of 1090 kN, and the overturning moment of 10900 kNm. The design should meet the CSA S304.1 Cl.10.16.5 requirements for shear walls of moderate ductility.

Use hollow concrete blocks of 20 MPa unit strength and Type S mortar. Consider the wall as solid grouted. Grade 400 steel reinforcement (yield strength f_y = 400 MPa) is used for this design.



SOLUTION:

1. Material properties

Steel (both reinforcing bars and joint reinforcement):

ϕ_s = 0.85 f_y = 400 MPa

Masonry:

ϕ_m = 0.6

S304.1 Table 4, 20 MPa concrete blocks and Type S mortar:

f'_m = 10.0 MPa (assume solid grouted masonry)

2. Load analysis

The section at the base of the wall needs to be designed for the following load effects:

- P_f = 1800 kN axial load

- V_f = 1090 kN seismic shear force
- M_f = 10900 kNm overturning moment

This is a moderate ductility shear wall, and NBCC 2005 Table 4.1.8.9 specifies the following R_d and R_o values (see Table 1-13):
R_d = 2.0 and R_o = 1.5
The seismic shear force of 1090 kN for a wall with moderate ductility ($R_d = 2.0$) was obtained by prorating the force of 1450 kN from Example 5a which corresponded to a shear wall with limited ductility ($R_d = 1.5$), as follows

$$V_f = 1450 * \frac{1.5}{2.0} \approx 1090 \text{ kN}$$

3. Height/thickness ratio check (Cl.10.16.5.2.2, see Section 2.5.4.4)
CSA S304.1-04 prescribes the following height-to-thickness (h/t) limit for the compression zone in <u>moderate</u> ductility shear walls:

$$h/(t+10) < 14$$

For this example,
h = 5000 mm (the largest unsupported wall height)
So,

$$t \geq h/14 - 10 = 347 \text{ mm}$$

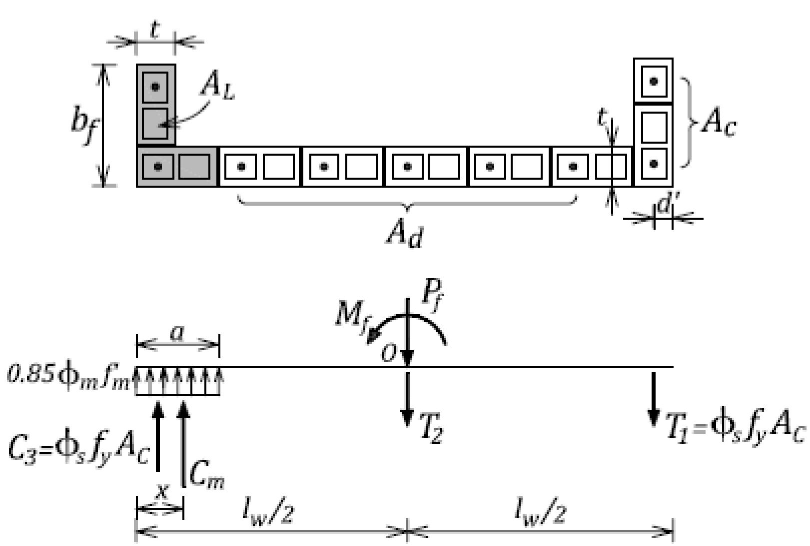
This exceeds the maximum possible wall thickness of 290 mm, which was used in Example 5a.

Commentary to Section 2.5.4.4 contains the following two alternative approaches for verifying the out-of-plane stability of ductile masonry shear walls:

1) Provide flanges at the wall ends and prove that the out-of-plane stability of the compression zone is satisfactory.
Try the effective flange width

$$b_f = 690 \text{ mm}$$

The wall section and the internal force distribution is shown on the figure.



This procedure assumes that the concentrated reinforcement (area A_c) is provided at the wall ends, while the remaining reinforcement (area A_d) is distributed over the wall length. After a few trial estimates, the total area of vertical reinforcement A_{vt} was determined as follows

A_{vt} = 2800 mm^2

Concentrated reinforcement area (2-15M bars at each wall end):

A_c = 400 mm^2

Distributed reinforcement area:

A_d = 2800-2*400=2000 mm^2

Distance from the wall end to the centroid of concentrated reinforcement A_c:

$d' = 145$ mm

- *Check the buckling resistance of the compression zone.*

The area of the compression zone A_L:

$$A_L = \frac{P_f + \phi_s f_y A_d}{0.85\phi_m f'_m} = \frac{1800*10^3 + 0.85*400*2000}{0.85*0.6*10.0} = 4.86*10^5$$

The depth of the compression zone a:

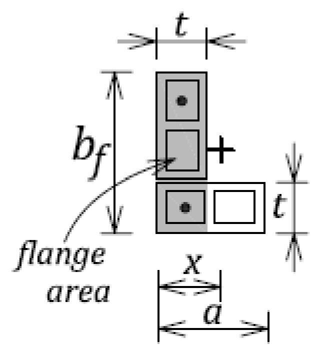
$$a = \frac{A_L - b_f * t + t^2}{t} = \frac{4.86*10^5 - (690*290) + 290^2}{290} = 1276 \text{ mm}$$

The neutral axis depth:

$$c = \frac{a}{0.8} = 1595 \text{ mm}$$

The centroid of the masonry compression zone:

$$x = \frac{t*\left(a^2/2\right) + (b_f - t)\left(t^2/2\right)}{A_L} = 520 \text{ mm}$$



In this case, the compression zone is L-shaped, however only the flange area will be considered for the buckling resistance check (see the shaded area shown on the figure). This is a conservative approximation and it is considered to be appropriate for this purpose, since the gross moment of inertia is used.

Gross moment of inertia for the flange only:

$$I_{xg} = \frac{t*b_f^{\,3}}{12} = \frac{290*690^3}{12} = 7.94*10^9 \text{ mm}^4$$

The buckling strength for the compression zone will be determined according to S304.1 Cl. 10.7.4.3, as follows:

$$P_{cr} = \frac{\pi^2 \phi_{er} E_m I}{(1+0.5\beta_d)(kH)^2} = 19983kN$$

where

$\phi_{er} = 0.75$

$k = 1.0$ pin-pin support conditions

$\beta_d = 0$ assume 100% seismic live load

$H = 5000$ mm unsupported wall height

$E_m = 850 f'_m = 8500$ MPa modulus of elasticity for masonry

- *Find the resultant compression force (including the concrete and steel component).*

$P_{fb} = C_m + \phi_s f_y A_c = 2480 * 10^3 + 0.85 * 400 * 400 = 2620$ kN

where

$C_m = (0.85 \phi_m f'_m) A_L = (0.85 * 0.6 * 10.0)(4.86 * 10^5) = 2480$ kN

- *Confirm that the out-of-plane buckling resistance is adequate.*

Since

$P_{fb} = 2620$ kN < $P_{cr} = 19983$ kN

it can be concluded that the out-of-plane buckling resistance is adequate. The flanged section can be used for this design.

2) Prove that the compression zone of the wall is small and the adjacent vertical strips are able to stabilize it.

For flanged wall sections, the neutral axis depth needs to meet one of the following requirement (see Figure 2-28c):

$c^* \leq 6t = 6 * 290 = 1740$ mm

Note that $6t$ denotes the distance from the inside of a wall flange to the point of zero strain. So, the total neutral axis depth (distance from the extreme compression fibre to the point of zero strain) is equal to

$c = c^* + t = 1740 + 290 = 2030$ mm

The neutral axis depth was determined above, as follows

$c = 1595$ mm < 2030 mm

Based on these two checks, the out-of-plane wall stability should not be a concern when a flanged section is used for the design, and so the CSA S304.1 height-to-thickness restrictions for moderate ductility shear walls will be relaxed.

4. Design the flanged section for the combined axial load and flexure – consider distributed and concentrated wall reinforcement (see Section C.1.1.1).

The key design parameters for this calculation were determined in step 3 above. The factored moment resistance M_r will be determined by summing up the moments around the centroid of the wall section as follows

$$M_r = C_m(l_w/2 - x) + 2(\phi_s f_y A_c)(l_w/2 - d') = 2480 * 10^3 * (10000/2 - 520) + 2 * (0.85 * 400 * 400) * (10000/2 - 145)$$

$M_r = 12431$ kNm > $M_f = 10900$ kNm   OK

5. Perform the CSA S304.1 ductility check (see Section 2.5.4.3).

To satisfy the CSA S304.1 ductility requirements for moderate ductility shear walls (Cl.10.16.5.2.3), neutral axis depth ratio (c/l_w) should be less than the following limit:

$c/l_w < 0.2$ when $h_w/l_w < 4$

In this case,

$h_w/l_w = 14000/10000 = 1.4 < 4$ and $c = 1595 mm$

thus

$c/l_w = 1595/10000 = 0.16 < 0.2$

Therefore, the CSA S304.1 ductility requirement is satisfied.

6. The diagonal tension shear resistance (see Section 2.3.2 and CSA S304.1 Cl.10.10.1)

Masonry shear resistance (V_m):

$b_w = 290$ mm overall wall thickness

$d_v \approx 0.8 l_w = 8000$ mm effective wall depth

$\gamma_g = 1.0$ solid grouted wall

$P_d = 0.9 P_f$ = 1620 kN

$v_m = 0.16(2 - \frac{M_f}{V_f d_v})\sqrt{f'_m}$ = 0.51 MPa

$\frac{M_f}{V_f d_v} = \frac{10900}{1090 * 8.0}$ = 1.25 > 1.0

$V_m = \phi_m (v_m b_w d_v + 0.25 P_d)\gamma_g$ = 0.6(0.51*290*8000+0.25*1620*10^3)*1.0 = 953 kN

To find the steel shear resistance V_s, assume 2-15M bond beam reinforcing bars at 600 mm spacing (this should make some allowance in the shear strength to satisfy capacity design), thus

$A_v = 400$ mm^2

$s = 600$ mm

$V_s = 0.6 \phi_s A_v f_y \frac{d_v}{s} = 0.6 * 0.85 * \frac{400}{1000} * 400 * \frac{8000}{600}$ = 1088 kN

According to Cl.10.16.5.3.1, there is a 50% reduction in the masonry shear resistance contribution for moderate ductility shear walls, and so

$V_r = 0.5 V_m + V_s = 0.5 * 953 + 1088 = 1565$ kN > $V_f = 1090$ kN OK

Maximum shear allowed on the section is (S304.1 Cl.10.10.1.1)

$\max V_r = 0.4 \phi_m \sqrt{f'_m} b_w d_v \gamma_g = 1760$ kN > V_r OK

7. Capacity design check (see Section 2.5.2)

At this point, both the moment resistance M_r and the diagonal shear resistance V_r for the wall section have been determined. S304.1 Cl.10.16.3.3 requires that ductile reinforced masonry shear walls be designed so that flexural failure takes place before shear failure has been initiated, that is, to follow the capacity design approach (see Section 2.5.2 for more details).

In this case, the factored moment resistance is equal to

$M_r = 12431$ kNm

The nominal moment resistance can be estimated as follows

$M_n = \frac{M_r}{\phi_s} = \frac{12431}{0.85} = 14625$ kNm

Shear force acts at the effective height h_e, that is, distance from the base of the wall to the resultant of all seismic forces acting at floor levels. h_e can be determined as follows

$h_e = \frac{M_f}{V_f} = 10.0$ m

The shear force V_{nb} that would cause the overturning moment equal to M_n is as follows

$$V_{nb} = \frac{M_n}{h_e} = \frac{14625}{10.0} = \text{ 1528 kN } < V_r = 1463 \text{ kN OK}$$

8. Shear resistance at the web-to-flange interface (Cl.7.11.4, see Section C.2).

The factored shear stress at the web-to-flange interface is equal to the larger of horizontal and vertical shear stress, as shown below.

Horizontal shear can be determined as follows:

$$v_f = \frac{V_f}{t_e l_w} = \frac{1090*10^3}{290*10000} = 0.38 \text{ MPa}$$

where t_e = 290 mm (effective wall thickness)

Vertical shear over the entire wall height (caused by the resultant compression force P_{fb} calculated in Step 3):

$$v_f = \frac{P_{fb}}{b_w * h_w} = \frac{2620*10^3}{290*14000} = 0.64 \text{ MPa} \qquad \text{governs}$$

Masonry diagonal tension shear strength:

$v_m = 0.51$ MPa (see step 6)

Since

$v_f = 0.64$ MPa > $\phi_m v_m = 0.31$ MPa

it is required to provide shear reinforcement at the web-to-flange interface. Since the horizontal reinforcement consists of 2-15M bars @ 600 mm spacing (bond beam reinforcement); both bars can be extended into the flange (90° hook), and so

$$v_s = \frac{\phi_s A_s f_y}{s \cdot t_e} = \frac{0.85*2*200*400}{600*290} = 0.78 \text{ MPa}$$

The total shear resistance

$v_r = \phi_m v_m + v_s = 0.31 + 0.78 = 1.09$ MPa > $v_f = 0.64$ MPa OK

9. Sliding shear resistance (see Section 2.5.4.6)

The factored in-plane sliding shear resistance V_r is determined as follows:

μ = 1.0 for a masonry-to-masonry or masonry-to-roughened concrete sliding plane

A_s = 2800 mm^2 total area of vertical wall reinforcement

For moderate ductility shear walls, only the vertical reinforcement in the tension zone should be accounted for in the T_y calculations (Cl.10.16.5.3.2), and so (see Figure 2-17b)

$$T_y = \phi_s A_s f_y \left(\frac{l_w - c}{l_w} \right) = 0.85*2800*400*\left(\frac{10000-1595}{10000} \right) = 800 \text{ kN}$$

P_d = 1620 kN

$P_2 = P_d + T_y$ = 1620+800 = 2420 kN

$V_r = \phi_m \mu P_2$ = 0.6*1.0*2420 = 1452 kN

$V_r = 1452$ kN > $V_f = 1090$ kN OK

10. CSA S304.1 seismic detailing requirements for moderate ductility walls – plastic hinge region

According to Cl.10.16.5.2.1, the required height of the plastic hinge region for moderate ductility shear walls must be greater than (see Table 2-4)

$l_p = l_w = 10.0$ m

or

$l_p = h_w / 6 = 14.0 / 6 = 2.3$ m

(note that h_w denotes the total wall height)

So, $l_p = 10.0$ m governs

Reinforcement detailing requirements for the plastic hinge region of limited ductility shear walls are as follows:

1. The wall in the plastic hinge region must be solid grouted (Cl.10.16.4.1.3, see Table 2-4).

2. Horizontal reinforcement requirements (see Figure 2-31)

a) Reinforcement spacing should not exceed the following limits (Cl.10.16.4.3.3), see Table 2-2:

$s \le 1200$ mm or

$s \le l_w/2 = 10000/2 = 5000$ m

Since the lesser value governs, the maximum permitted spacing is

$s \le 1200$ mm

According to the design (see step 7), the horizontal reinforcement spacing is 600 mm, hence OK.

b) Detailing requirements (see Table 2-3)

Horizontal reinforcement shall not be lapped within (Cl.10.16.4.3.3)

600 mm or

c = 1595 mm (the neutral axis depth)

whichever is greater, from the end of the wall. In this case, the reinforcement should not be lapped within the distance c = 1595 mm from the end of the wall. The horizontal reinforcement can be lapped at the wall half-length.

Horizontal reinforcement shall be (Cl.10.16.5.4.2):

i) provided by reinforcing bars only (no joint reinforcement!);

ii) continuous over the length of the wall (can be lapped in the centre), and

iii) have 180° hooks around the vertical reinforcing bars at the ends of the wall.

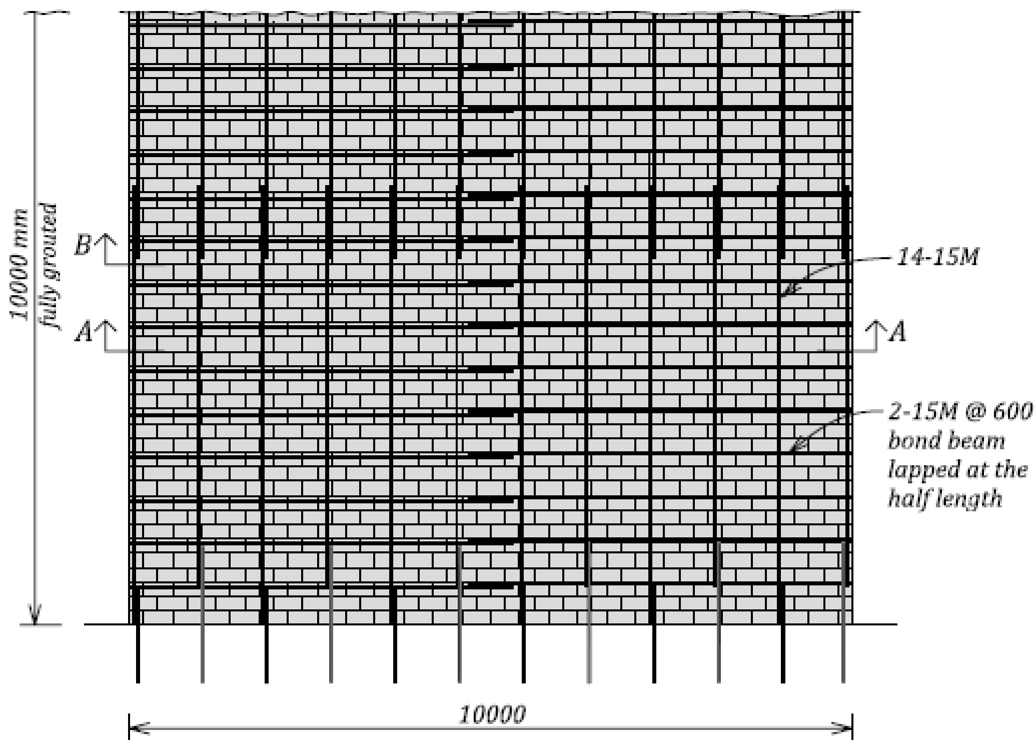
All these requirements will be complied with, as shown on the design summary drawing.

3. Vertical reinforcement requirements (Cl.10.16.5.4.1)

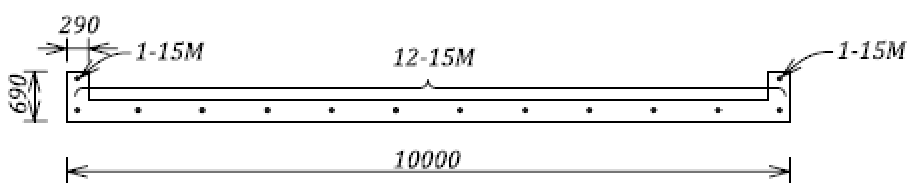
At any section within the plastic hinge region, no more than half of the area of vertical reinforcement may be lapped (see Table 2-3 and Figure 2-31).

11. Design summary

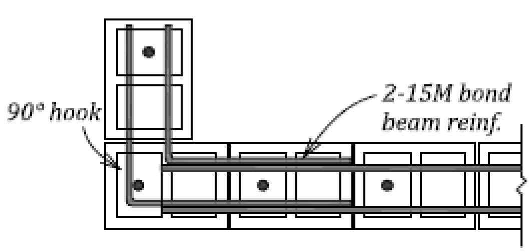
Reinforcement arrangement for the wall under consideration is summarized on the next page. Note that the shear wall of limited ductility must be solid grouted in plastic hinge region, but it may be partially grouted outside the plastic hinge region (this depends on the design forces).



Elevation



Section A-A



Section B-B

12. Discussion

It is important to consider all possible behaviour modes and identify the one that governs in this design. The following three shear resistance values need to be considered:

a) $V_{nb} = 1463$ kN shear force corresponding to flexural failure

b) $V_r = 1565$ kN diagonal tension shear resistance

c) $V_r = 1452$ kN sliding shear resistance

Since the sliding shear resistance value is smallest, it can be concluded that the sliding shear mechanism is critical in this case. Sliding shear resistance can be increased by roughening the wall-to-foundation interface (in which case the frictional coefficient can be increased to μ = 1.4) or by providing shear keys. Alternatively, additional dowels could be provided at the base of the wall, however this would result in an increase in the moment resistance. The designer would need to ensure that the capacity design criterion discussed in step 7 is satisfied.

At this point, it is of interest to compare the designs for limited ductility shear wall (Example 5a) and the moderate ductility shear wall (Example 5b). The walls are very similar, and have the same height, length, and thickness. The wall from this example has a flanged section, while the wall from Example 5a has a rectangular section. The walls are subjected to the same seismic hazard, but differ depending on the R_d and R_o values required for the respective designs. The comparison of the two designs is presented in the table below.

Table 1. A Comparison of the Limited Ductility and Moderate Ductility Shear Wall Designs

	Limited ductility shear wall (Example 5a)	**Moderate ductility shear wall (Example 5b)**
Actual height/thickness ratio for the plastic hinge zone (CSA S304.1 limit)	17.2 (18)	17.2 (14) – flanges used to stabilize compression zones at wall ends
Vertical reinforcement	6000 mm^2 (20-20M)	2800 mm^2 (14-15M)
Horizontal reinforcement	2-15M@800 mm bond beam reinforcement	2-15M@600 mm bond beam reinforcement
Plastic hinge length l_p	5 m	10 m
Horizontal reinforcement detailing (Table 2-3)	Minimal requirements	More extensive detailing requirements (see step 10)
Vertical reinforcement detailing (Table 2-3)	No special requirements	Lapping requirement (see step 10)

The key differences in these designs can be summarized as follows:

1. Moderate ductility shear wall requires less vertical reinforcement (by approximately 50%) since the dead load provides proportionally more of the moment resistance.

2. Moderate ductility shear wall requires more horizontal reinforcement (by approximately 30%) as only 50% of the masonry shear strength can be utilized.

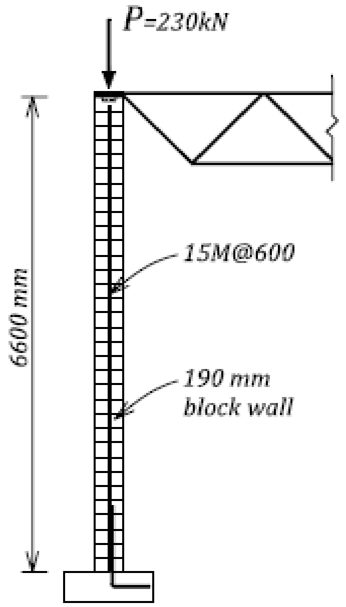
3. Moderate ductility shear wall requires a substantially larger plastic hinge zone (10 m high) as compared to the limited ductility shear wall (5 m high) – the wall needs to be solid grouted and special detailing requirements apply in this zone, although the limited ductility shear wall may also need to be solid grouted for a height greater than 5 m.

4. Moderate ductility shear wall has more extensive horizontal and vertical reinforcement detailing requirements.

5. Moderate ductility shear wall requires flanges in order to satisfy the CSA S304.1 height/thickness requirements.

<u>EXAMPLE 6 a</u>: Design of a loadbearing wall for out-of-plane seismic effects

Verify the out-of-plane seismic resistance of the loadbearing block wall designed for in-plane loads in Example 4b, according to NBCC 2005 and CSA S304.1 requirements. The wall is a part of a single-storey warehouse building located in Ottawa, ON, with soil corresponding to Site Class C. The wall is 8 m long and 6.6 m high, and is subjected to a total dead load of 230 kN (including its self-weight). The wall is constructed with 200 mm hollow concrete blocks of 15 MPa unit strength, Type S mortar, and solid grouting. The wall is reinforced with 15M Grade 400 vertical rebars at 600 mm on centre spacing. The slenderness effects outlined in CSA S304.1 will not be considered in this design.



<u>SOLUTION:</u>

1. Material properties

Steel (both reinforcing bars and joint reinforcement):

ϕ_s = 0.85 f_y = 400 MPa

Masonry:

ϕ_m = 0.6

S304.1 Table 4, 15 MPa concrete blocks and Type S mortar:

f'_m = 7.5 MPa (assume solid grouted masonry)

2. Determine the out-of-plane seismic load according to NBCC 2005 (see Section 2.6.5.3).

This design requires the calculation of seismic load V_p for parts of buildings and nonstructural components according to NBCC 2005 Cl.4.1.8.17. First, seismic design parameters need to be determined as follows:

- Location: Ottawa, ON

  $S_a(0.2)$ = 0.66 (NBCC 2005 Appendix C, page C-22)

- Foundation factors

  F_a = 1.0 for $S_a(0.2)$ =0.66 and Site Class C (from Table 1-10 or NBCC 2005 Table 4.1.8.4.B)
- I_E = 1.0 normal importance building

Find S_p (horizontal force factor for part or portion of a building and its anchorage per NBCC 2005, Table 4.1.8.17, Case 1)

$C_p = 1.0 \quad A_r = 1.0 \quad R_p = 2.5 \quad A_x = 3.0$ ($h_x = h_n$ top floor)

$S_p = C_p A_r A_x / R_p = 1.0 \cdot 1.0 \cdot 3.0/2.5 = 1.2$

$0.7 < S_p < 4.0$ O.K.

- W_p = 4.0 kN/m^2 unit weight of the 190 mm block wall (solid grouted)

Seismic load V_p can be calculated as follows:

$V_p = 0.3 F_a S_a(0.2) I_E S_p W_p$=0.3*1.0*0.66*1.0*1.2*(4.0 kN/m^2) = 0.95 kN/m^2 ≈ 1.0 kN/m^2

3. Determine the effective compression zone width (b) for the out-of-plane design (see Section 2.4.2).

According to S304.1 Cl.10.6.1, the effective compression zone width (b) should be taken as the lesser of the following two values (see Figure 2-19):

$b = s = 600$ mm spacing of vertical reinforcement

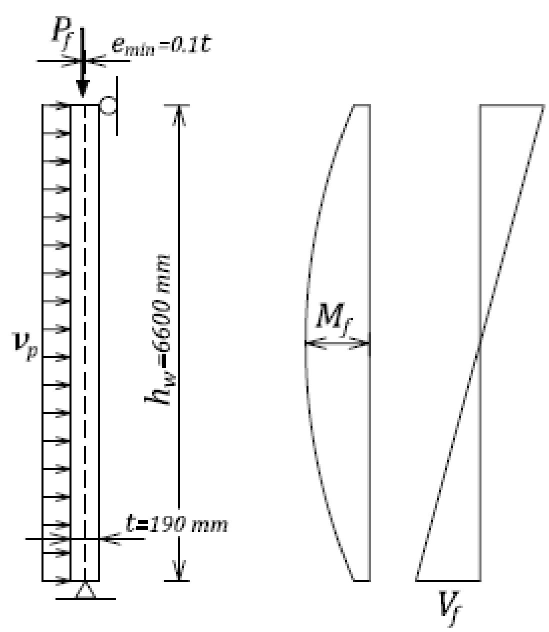
or

$b = 4t = 4*190 = 760$ mm

All design calculations in this example will be performed considering a vertical wall strip of width $b = 600$ mm.

4. Find the design shear force and the bending moment.

The wall will be modeled as a simple beam with pin supports at the base and top. The loads on the wall consist of axial load due to roof load and wall self-weight, plus the seismic out-of-plane load. The roof load and wall self-weight create moments due to minimum axial load eccentricity.



- Axial load per wall width equal to $b = 600$ mm:

$P_f = \frac{P}{l_w} * b = \frac{230kN}{8m} * 0.6 = 17.25 \approx 17.0$ kN

- Minimum eccentricity (S304.1 Cl.10.7.2)

$e_{\min} = 0.1t =$ 0.019 m

- Out-of-plane seismic load per wall width equal to $b = 600$ mm:

$v_p = 1.0 * 0.6 = 0.6$ kN/m

- Design bending moment (at the midheight):

$M_f = p * e_{\min} + \frac{v_p * h_w^2}{8} = 17 * 0.019 + \frac{0.6 * 6.6^2}{8}$

= 3.59 ≈ 3.6 kNm

5. Check whether the wall resistance for the combined effect of axial load and bending is adequate (see Section C.1.2).
This can be verified from a P-M interaction diagram which can be developed using the EXCEL© software (or commercially available masonry design software). Relevant tables used to develop the diagram are presented below, while the detailed theoretical background is outlined in Section C.1.2. Note that the design width is equal to $b = 600mm$.

Table 1. Design Parameters

Design parameter	Unit	Symbol	Value
Wall thickness	mm	t	190
Design width	mm	b	600
Masonry maximum strain		EPSm	0.003
Masonry strength	MPa	f'm	7.5
Steel yield strength	MPa	fy	400
Steel modulus of elasticity	MPa	Es	200000
Effective depth	mm	d	95
(c/d)balanced			0.6
Reinforcement area	mm^2/b	As	200
Material resistance-masonry		Fim	0.6
Material resistance-steel		Fis	0.85
X- factor		X	1
BETA1		BETA1	0.8
Effective area	mm^2	Ae	114000

In this case, the reinforcement is placed at the centre of the wall and so

$d = \frac{t}{2} = \frac{190}{2} = 95$ mm

The neutral axis depth corresponding to a balanced condition (onset of yielding in the steel and maximum compressive strain in masonry) can be determined from the following proportion

$$\frac{c_b}{d - c_b} = \frac{\varepsilon_m}{\varepsilon_y}$$

For $\varepsilon_m = 0.003$ and $\varepsilon_y = 0.002$ it follows that

$c_b = 0.6d$

The area of vertical reinforcement per width $b = 600$ mm can be determined as follows:

$A_s = \frac{A_b}{s} * b = \frac{200}{600} * 600 = 200$ mm^2 (15M@ 600 mm reinforcement)

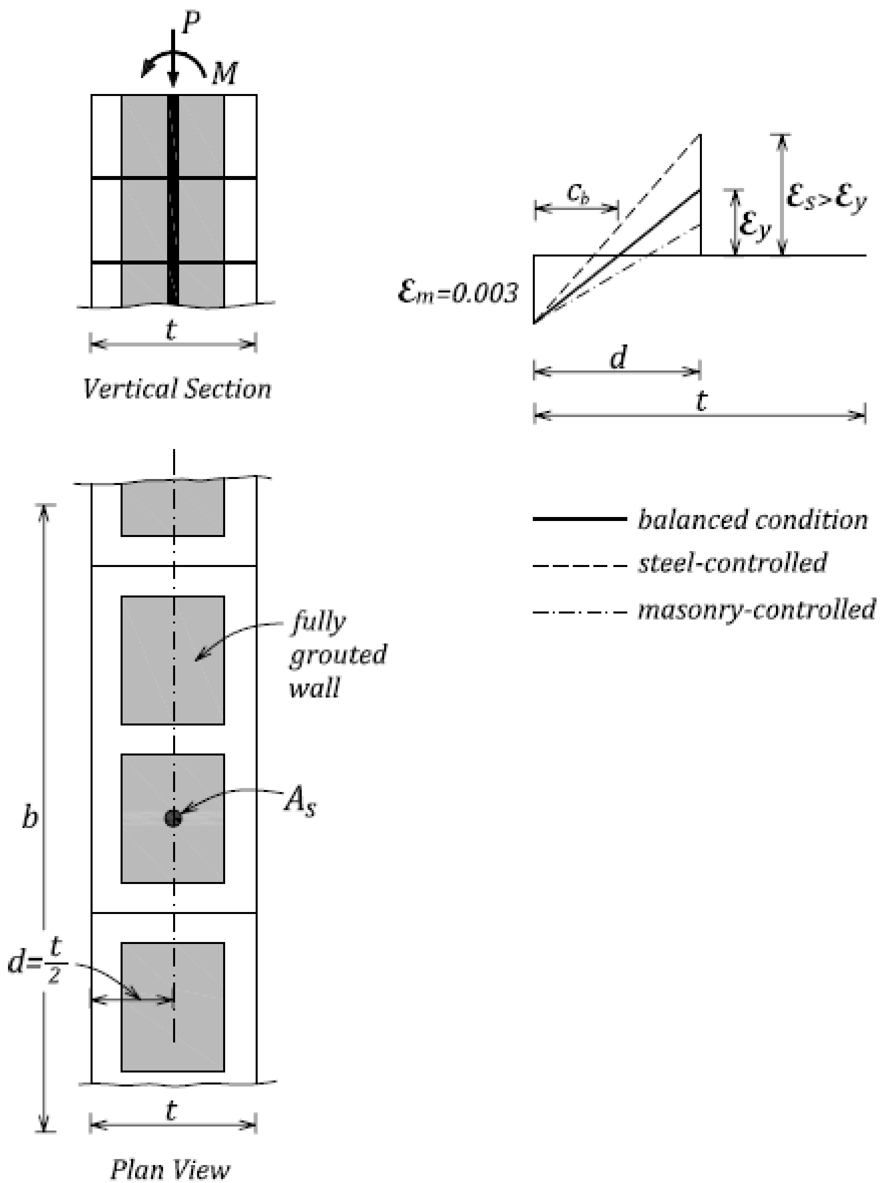
To determine whether the wall can carry the combined effect of axial load and bending moment, it is useful to construct an axial load-moment interaction diagram (also known as P-M interaction diagram). The P-M interaction diagram for this example was developed using Microsoft

EXCEL® spreadsheet, but other methods or computer programs are also available. The results of the calculations are presented in Table 2.

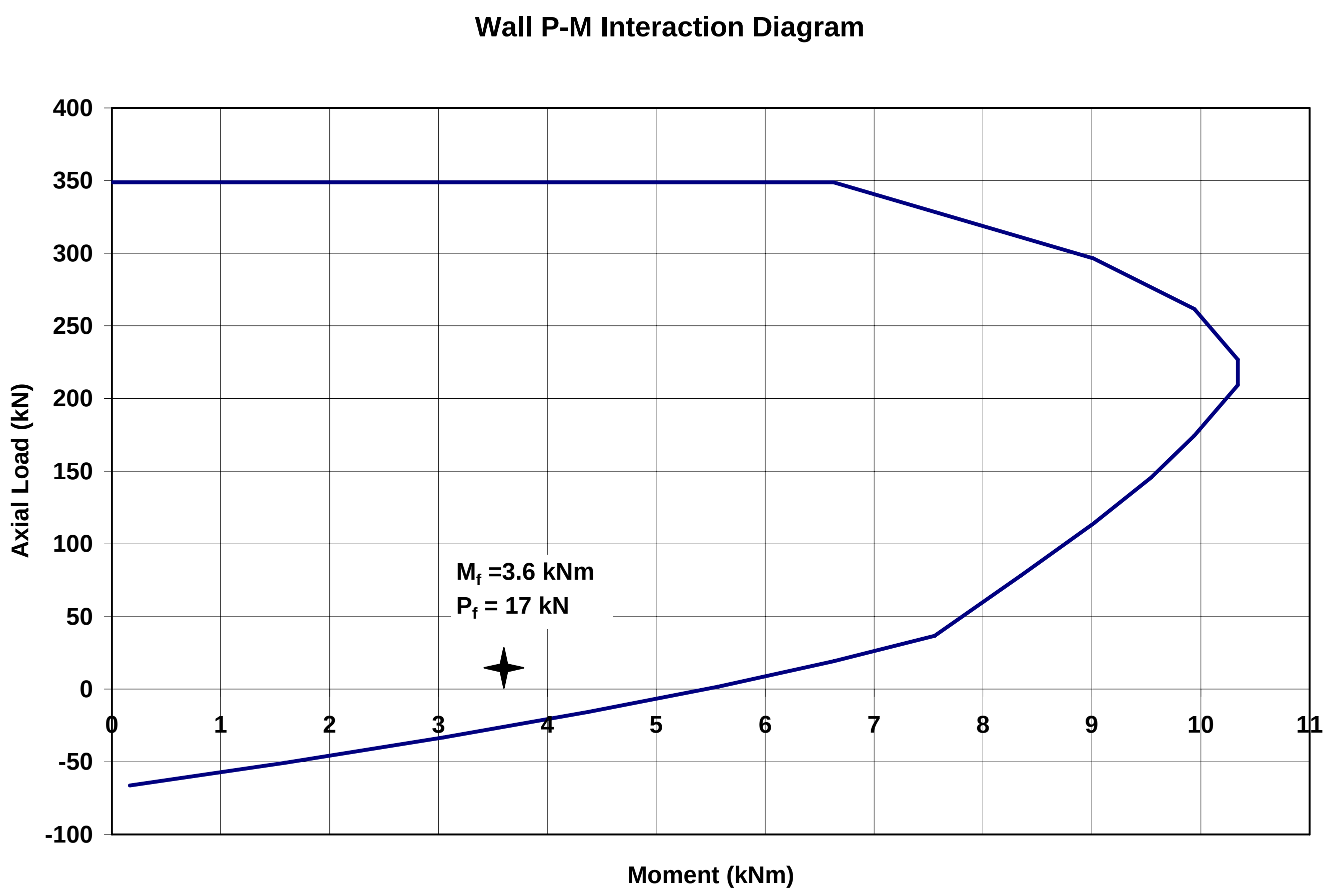
Table 2. P-M Interaction Diagram Values

		c/d	c	C_m	EPSs	T_r	M_r	P_r
			mm	N		N	kNm	kN
Points controlled by steel $c<c_b$		0.01	0.95	1744.2	0.02	68000	0.16504	-66.256
		0.1	9.5	17442	0.02	68000	1.59071	-50.558
		0.2	19	34884	0.02	68000	3.04886	-33.116
		0.3	28.5	52326	0.02	68000	4.37445	-15.674
		0.4	38	69768	0.02	68000	5.56749	1.768
		0.5	47.5	87210	0.02	68000	6.62796	19.21
		0.6	57	104652	0.02	68000	7.55587	36.652
Points controlled by masonry $c>c_b$		0.6	57	104652	0.002	68000	7.55587	36.652
		0.7	66.5	122094	0.00129	43714.3	8.35123	78.3797
		0.8	76	139536	0.00075	25500	9.01403	114.036
		0.9	85.5	156978	0.00033	11333.3	9.54426	145.645
Full section under compression		1	95	174420	0	0	9.94194	174.42
		1.2	114	209304	-0.0005	-17000	10.3396	209.304
		1.3	123.5	226746	-0.0007	-23538	10.3396	226.746
		1.5	142.5	261630	-0.001	-34000	9.94194	261.63
		1.7	161.5	296514	-0.0012	-42000	9.01403	296.514
		2	190	348840	-0.0015	-51000	6.62796	348.84
Pure compression							0	348.84

The three basic cases considered in the development of the interaction diagram (steel-controlled behaviour, masonry-controlled behaviour, and the balanced condition) are illustrated on the figure below. For more detailed explanation related to the development of P-M interaction diagrams refer to Section C.1.2.



The P-M interaction diagram showing the point of interest ($M_f = 3.6$ kNm and $P_f = 17$ kN) is shown below. It is obvious that the wall resistance to combined effects of axial load and out-of-plane bending is adequate for the given design loads and the reinforcement determined in Example 4b.



6. Check whether the out-of-plane shear resistance of the wall is adequate (Cl.10.10.2, see Section 2.4.2).

Design shear force at the support per wall width $b = 600$ mm:

$$V_f = \frac{v_p * h_w}{2} = \frac{0.6 * 6.6}{2} \approx 2.0 \text{ kN}$$

According to S304.1 Cl.10.10.2, the factored out-of-plane shear resistance (V_r) shall be taken as follows

$$V_r = \phi_m (v_m \cdot b \cdot d + 0.25 P_d)$$

where

$v_m = 0.16\sqrt{f'_m}$ = 0.44 MPa (f'_m = 7.5 MPa for solid grouted 15 MPa block)

$d = 95$ mm effective depth (to the block mid-depth)

$b = 600$ mm effective compression zone width

The axial load P_d can be determined as

$$P_d = 0.9 P_f = 0.9 * 17.25 = 15.5 \text{ kN}$$

(note that the load has been prorated in proportion to the effective compression zone width b).

So,

$$V_r = 0.6 * (0.44 * 600 * 95 + 0.25 * 15500) = 17.4 \text{ kN}$$

Since

$V_f = 2.0$ kN < $V_r = 17.4$ kN   OK

Maximum shear allowed on the section is

$\max V_r = 0.4\phi_m \sqrt{f'_m}\left(b * d\right) = 0.4 * 0.6 * \sqrt{7.5} * (600 * 95) = 37.5$ kN   OK

7. Check the sliding shear resistance (see Section 2.4.3).

The factored out-of-plane sliding shear resistance V_r is determined according to S304.1 Cl.10.10.4.2, as follows:

μ = 1.0 for a masonry-to-masonry or masonry-to-roughened concrete sliding plane

A_s = 200 mm^2 area of vertical reinforcement per wall width $b = 600$ mm

$T_y = \phi_s A_s f_y$ = 0.85*200*400 = 68 kN

$P_d = 0.9 P_f = 15.5$ kN

$P_2 = P_d + T_y$ = 15.5+68 = 83.5 kN

$V_r = \phi_m \mu P_2$= 0.6*1.0*83.5= 50.0 kN

$V_r = 50.0$ kN > $V_f = 2.0$ kN   OK

Note that the sliding shear resistance does not govern in this case, however this mechanism often governs the in-plane shear resistance.

8. Conclusion

It can be concluded that the out-of-plane seismic resistance for this wall is satisfactory. This wall seems to be overdesigned for the out-of-plane resistance because the in-plane seismic design governs (this is a common scenario in design practice).

<u>EXAMPLE 6 b</u>: Design of a nonloadbearing wall for out-of-plane seismic effects

Consider the same masonry wall discussed in Example 6a, but in this example treat is as a nonloadbearing wall. The wall is 8 m long and 6.6 m high and is constructed using 200 mm hollow concrete blocks of 15 MPa unit strength and Type S mortar. Verify the out-of-plane seismic resistance of the wall according to NBCC 2005 and CSA S304.1 seismic requirements.

Consider the following two cases:
a) unreinforced wall, and
b) reinforced partially grouted wall (use Grade 400 steel reinforcement for this design).

Use the seismic load determined in Example 6a, that is, $v_p = 1.0$ kN/m^2.

<u>SOLUTION:</u>

Material properties
Steel (both reinforcing bars and joint reinforcement):
ϕ_s = 0.85 f_y = 400 MPa
Masonry:
ϕ_m = 0.6
Compresion resistance (S304.1 Table 4, 15 MPa concrete blocks and Type S mortar):
f'_m = 9.8 MPa (ungrouted, or partially grouted ignoring grout area)
Tension resistance normal to bed joint (S304.1 Table 5):
f_t = 0.4 MPa (ungrouted)

Find the design shear force and the bending moment.
The wall will be modeled as a simple beam with pin supports at the base and the top. The wall height is $h_w = 6.6$ m. A unit wall strip (width $b = 1000$ mm) will be considered for this design.

The forces on the wall consist of the axial load due to the wall self-weight and the bending moment due to seismic out-of-plane load (NBCC 2005 load combination 1xD+1xE).

- Factored axial load per width b of 1.0 m:

wall weight w = 2.46 kN/m^2 (ungrouted 190 mm block wall)

$$P_f = w * \frac{h_w}{2} * b = (2.46) * \frac{6.6}{2} * 1.0 = 8.1 \text{ kN/m}$$

- Out-of-plane seismic load per width b of 1.0 m:

$v_p = 1.0$ kN/m

- Factored bending moment (at the midheight):

$$M_f = \frac{v_p * h_w^{\ 2}}{8} = \frac{1.0 * 6.6^2}{8} \approx 5.5 \text{ kNm/m}$$

- Factored shear force (at the support):

$$V_f = \frac{v_p * h_w}{2} = \frac{1.0 * 6.6}{2} \approx 3.3 \text{ kN/m}$$

a) Unreinforced wall

Check whether the wall resistance to the combined effect of axial load and bending is adequate (see Section 2.6.1.3).

Find the load eccentricity:

$$e = \frac{M_f}{P_f} = \frac{5.5kNm}{8.1kN} = 0.68m = 680mm$$

According to S304.1 Cl.7.2.1, an unreinforced masonry wall is to be designed as uncracked if

$e > 0.33t$

where t denotes the wall thickness ($t = 190mm$)

$0.33t = 0.33*190 = 63mm$

In this case,

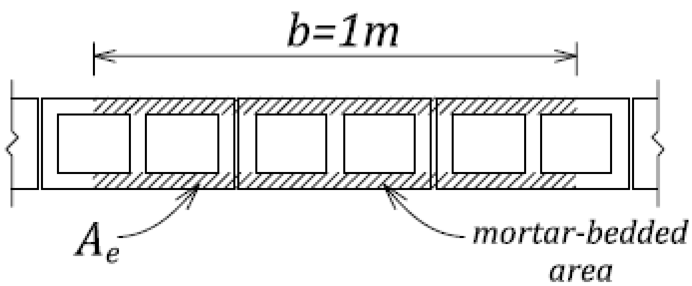
$e = 680mm > 0.33t = 63mm$

so the wall will be designed as uncracked (i.e. the maximum tensile stress is less than the allowable value) according to S304.1 Cl.7.2.2. The design procedure is explained in Section 2.6.1.3.

First, we need to determine properties for the effective wall section for a width $b = 1000$ mm. For a hollow 190 mm wall, the values obtained from Table D-1 are as follows:

$A_e = 75.4*10^3$ mm^2/m effective cross-sectional area

$S_e = 4.66*10^6$ mm^3/m section modulus of effective cross-sectional area



The maximum compression stress at the wall face can be calculated as follows:

$$\max f_c = \frac{P_f}{A_e} + \frac{M_f}{S_e} = \frac{8.1*10^3}{75.4*10^3} + \frac{5.5*10^6}{4.66*10^6} = 0.107 + 1.18 = 1.29MPa$$

The allowable value is equal to

$\phi_m f'_m = 0.6*9.8 = 5.9MPa$

Since

$\max f_c = 1.29MPa < 5.9MPa$

it follows that the maximum compression stress is less than the allowable value.

Find the maximum tensile stress as follows:

$$\max f_t = \frac{P_f}{A_e} - \frac{M_f}{S_e} = \frac{8.1*10^3}{75.4*10^3} - \frac{5.5*10^6}{4.66*10^6} = 0.107 - 1.18 = -1.07MPa$$

The allowable value is equal to

$-\phi_m f_t = -0.6*0.4 = -0.24MPa$

Since

$\max f_t = -1.07 MPa < -0.24 MPa$

it follows that the maximum tensile stress exceeds the allowable value, which is not acceptable.

In this design, the tensile stress criterion is not going to be satisfied even if the wall thickness is increased to 290 mm. Therefore, a reinforced masonry wall is required in this case. Also, reinforcement in this wall is mandatory since the wall is to be constructed at Ottawa, ON, where the seismic hazard index $I_E F_a S_a(0.2)$=1.0*1.0*0.66=0.66 > 0.35. Therefore, the design will proceed considering a reinforced nonloadbearing wall.

b) Reinforced wall

1. Find the minimum seismic reinforcement for nonloadbearing walls (see Section 2.6.4).

According to S304.1 Cl.10.15.2.4, if $0.35 \le I_E F_a S_a(0.2) \le 0.75$ nonloadbearing walls shall be reinforced in one or more directions with reinforcing steel having a minimum total area of

$A_{stotal} = 0.0005 A_g$

The reinforcement may be placed in one direction, provided that it is located to reinforce the wall adequately against lateral loads and spans between lateral supports.

$A_{stotal} = 0.0005 A_g$ = 0.0005*(190*10^3 mm^2) = 95 mm^2/m

where

A_g =(1000mm)*(190mm)=190*10^3 mm^2 gross cross-sectional area per metre of wall length

Let us choose 15M vertical reinforcement (area 200 mm^2) at 1200 mm spacing which is the maximum spacing allowed (1200 mm).

The area of reinforcement per metre of wall length is

$A_s = 200 * \frac{1000}{1200} = 167$ mm^2/m > 95 mm^2/m OK

2. Determine the effective compression zone width (b) for the out-of-plane design (see Section 2.4.2).

The wall resistance will be determined considering a strip equal to the bar spacing s =1200 mm, as follows:

$P_f = 8.1 * \frac{1.2}{1.0} = 9.7$ kN

$M_f = 5.5 * \frac{1.2}{1.0} = 6.6$ kNm

$V_f = 3.3 * \frac{1.2}{1.0} = 4.0$ kN

3. Check whether the wall resistance to the combined effect of axial load and bending is adequate (see Section C.1.2).

Since this is a partially grouted wall, its flexural resistance will be determined using a T-section model.

According to S304.1 Cl.10.6.1, the effective compression zone width (b) should be taken as the lesser of the following two values (see Figure 2-19):

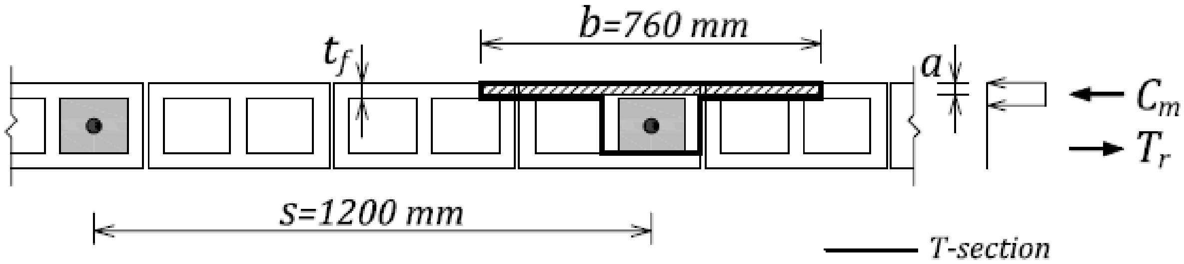
$b = s = 1200$ mm

or

$b = 4t = 4*190 = 760$ mm

Therefore, $b = 760$ mm will be used as the width of the masonry compression zone.

A typical wall cross-section is shown on the figure below. Note that the face shell thickness is 38 mm (typical for a hollow block masonry unit). The same value can be obtained from Table D-1, considering the case of an ungrouted 200 mm block wall.



Since the reinforcement is placed at the centre of the wall, the effective depth is equal to

$$d = \frac{t}{2} = \frac{190}{2} = 95 \text{ mm}$$

The reinforcement area used for the design needs to be determined as follows:

$$A_s = A_b = 200 \text{ mm}^2$$

The internal forces will be determined as follows (see Figure C-8):

$$T_r = \phi_s f_y A_s = 0.85 * 400 * 200 = 68000 \text{ N}$$

Since

$$C_m = P_f + T_r = 9700 + 68000 = 77700 \text{ N}$$

and

$$C_m = (0.85\phi_m f'_m)(b \cdot a)$$

the depth of the compression stress block a can be determined as follows

$$a = \frac{C_m}{0.85\phi_m f'_m b} = \frac{77700}{0.85 * 0.6 * 9.8 * 760} = 20 \text{ mm}$$

Since

$$a = 20mm < t_f = 38mm$$

the neutral axis is located in the face shell (flange). The moment resistance around the centroid of the wall section can be determined as follows

$$M_r = C_m (d - a/2) = 77700 * (95 - 20/2) = 6.6 \text{ kNm}$$

Since

$$M_r = 6.6 \text{ kNm} = M_f = 6.6 \text{ kNm}$$

it follows that the wall flexural resistance is adequate. However, the reinforcement spacing could be reduced to s =1000 mm to allow for an additional safety margin (the revised moment resistance calculations are omitted from this example).

4. Check whether the out-of-plane shear resistance of the wall is adequate (see Section 2.4.2).

According to S304.1 Cl.10.10.2, the factored out-of-plane shear resistance (V_r) shall be taken as follows

$$V_r = \phi_m (v_m \cdot b \cdot d + 0.25 P_d) \qquad \text{where}$$

$v_m = 0.16\sqrt{f'_m}$ = 0.50 MPa

$d = 95$ mm effective depth

$b \approx 200$ mm web width - equal to the grouted cell width (156 mm) plus the thickness of the adjacent webs (26 mm each)

The axial load P_d can be determined as

$P_d = 0.9P_f = 0.9*9.7 = 8.7$ kN

Thus,

$V_r = 0.6*(0.50*200*95 + 0.25*8700) = 7.0$ kN

Since

$V_f = 4.0$ kN < $V_r = 7.0$ kN OK

Maximum shear allowed on the section is

$\max V_r = 0.4\phi_m\sqrt{f'_m}(b*d) = 0.4*0.6*\sqrt{9.8}*(200*95) = 14.3$ kN OK

5. Check the sliding shear resistance (see Section 2.4.3).

The factored in-plane sliding shear resistance V_r is determined according to S304.1 Cl.10.10.4.2, as follows:

μ = 1.0 for a masonry-to-masonry or masonry-to-roughened concrete sliding plane

A_s = 200 mm^2 area of vertical reinforcement at 1.2 m spacing

$T_y = \phi_s A_s f_y$ = 0.85*200*400 = 68.0 kN

P_d = 8.7 kN

$P_2 = P_d + T_y$ = 8.7+68.0 = 76.7 kN

$V_r = \phi_m \mu P_2$ = 0.6*1.0*76.7 = 46.0 kN

$V_r = 46.0$ kN > $V_f = 4.0$ kN OK

6. Conclusion

It can be concluded that the out-of-plane seismic resistance of this nonloadbearing wall is satisfactory. It should be noted that the flexural resistance governs in this design. The required amount of vertical reinforcement (15M@1200 mm) corresponds to the following area per metre length

$$A_s = A_b * \frac{1000}{s} = 167 \text{ mm}^2$$

which is significantly larger than the minimum seismic reinforcement prescribed by CSA S304.1, that is, $A_{stotal} = 95$ mm^2/m. Note that 15M@1200 mm is also the minimum vertical reinforcement that meets the minimum spacing requirements using typical15M bars.

Also, since horizontal reinforcement does not contribute to out-of-plane wall resistance, it was not considered in this example. However, provision of 9 Ga. horizontal ladder reinforcement at 400 mm spacing could be considered to improve the overall seismic performance of the wall.

It should be noted that, in exterior walls the mortar-bedded joints could be significantly affected by the presence of aesthetic joint finishes characterized by deeper grooves (e.g. raked joints); some of the grooves are up to 10 mm deep. The designer should consider this effect in the calculation of the compression zone depth.

EXAMPLE 7: Seismic design of masonry veneer ties

Perform the seismic design for tie connections for a 4.8 m high concrete block veneer wall in a school gymnasium in Montréal, Quebec. The building is founded on rock. The design should be performed to the requirements of NBCC 2005, CSA S304.1-04, and CSA A370-04. Consider the following two types of the veneer backup:
a) Concrete block wall (a rigid backup), and
b) Steel stud wall with 400 mm steel stud spacing (a flexible backup).
c) Evaluate the minimum tie strength requirements for the rigid and flexible backup.

SOLUTION:

This design problem requires the calculation of seismic load V_p for nonstructural elements according to NBCC 2005 Cl.4.1.8.17 (for more details see Section 2.6.5.3). Note that the wind load could govern in a tie design for many site locations in Canada, however wind load calculations were omitted for this seismic design example.

First, seismic design parameters need to be determined as follows:

- Location: Montréal, Quebec
  $S_a(0.2)$ = 0.69 (NBCC 2005 Appendix C, page C-26)
- Foundation factors
  Site Class = B (rock)
  F_a = 0.88 for $S_a(0.2)$ =0.69 and Site Class B (by interpolation from Table 1-10 or NBCC 2005 Table 4.1.8.4.B), since
  F_a = 0.8 for $S_a(0.2) =$ 0.50
  F_a = 0.9 for $S_a(0.2) =$ 0.75
- I_E = 1.3 school (high importance building)

At this point, it would be appropriate to check whether the seismic design of ties is required for this design. According to NBCC 2005 Cl.4.1.8.17.2, seismic design of ties is required when the seismic hazard index $I_E F_a S_a(0.2) \geq 0.35$ (and also for post-disaster buildings in lower seismic regions).
In this case,
$I_E F_a S_a(0.2)$= 1.3*0.88*0.69=0.79 $\geq$ 0.35
Therefore, seismic design is required.

- Find S_p (horizontal force factor for part or portion of a building and its anchorage per NBCC 2005, Table 4.1.8.17, Case 8)

$S_p = C_p A_r A_x / R_p = 1.0 \cdot 1.0 \cdot 3.0/1.5 = 2.0$
where
$A_x = 1 + 2h_x/h_n = 3.0$ for top of wall worst case
Since $0.7 < S_p < 4.0$ O.K.

- W_p = 1.8 kN/m^2 unit weight of the veneer masonry (concrete blocks)

Seismic load V_p can be calculated as follows:
$V_p = 0.3 F_a S_a(0.2) I_E S_p W_p$=0.3*0.88*0.69*1.3*2.0*(1.8 kN/m^2) =0.85 kN/m^2
Note that the above load is determined per m^2 of the wall surface area.

a) Concrete block backup (rigid)
Assume the maximum tie spacing permitted according to CSA S304.1 Cl.9.1.3 of 600 mm vertically and 820 mm horizontally (see Section 2.6.5.2), resulting in a tributary tie area for a concrete backup wall of
A = 0.82*0.60 = 0.49 m^2
The required factored tie capacity should exceed the factored tie load, that is,
$V_f \geq V_p * A$ = (0.85 kN/m^2)*(0.49 m^2) = 0.42 kN
Alternatively, for a given tie capacity, a tie spacing could be determined based on the maximum tributary area calculated from V_p and the factored tie capacity V_f , that is,
$A \leq V_f / V_p$

b) Steel stud backup (flexible)
Since the steel stud is a flexible backup, a tie must be able to resist 40% of the tributary lateral load on a vertical line of ties (S304.1 Cl.9.1.3.3, see Section 2.6.5.3):
$V_f \geq 0.4 * V_p * A_t$ = 0.4*(0.85 kN/m^2)*(1.92m^2) = 0.65 kN
where A_t = 0.4m*4.8m = 1.92 m^2 is tributary area on a vertical line of ties based on a probable 0.4 m horizontal tie spacing, and 4.8 m wall height

According to the same S304.1 clause, the tie must also be able to resist a load corresponding to double the tributary area on a tie, that is,
$V_f = 2 * V_p * A$ = 2*(0.85 kN/m^2)*(0.4m*0.6m) = 0.41 kN
Note that the tributary area was based on a 0.4 m stud spacing, and the maximum vertical tie spacing of 0.6 m prescribed by S304.1 Cl. 9.1.3.1.

In conclusion, the tie design load for the flexible veneer backup is V_f = 0.65 kN.

c) Minimum strength requirements
CSA A370-04 Cl.8.1 prescribes minimum <u>ultimate</u> tensile/compressive tie strength of 1 kN. In order to obtain the ultimate tie strength, the factored strength needs to be divided by the resistance factor ϕ. According to CSA A370-04 Cl.9.4.2.1.2, the resistance factor is 0.9 for tie material strength, or 0.6 for embedment failure, failure of fasteners, or buckling failure of the connection. It is conservative to use lower resistance factor in determining the ultimate tie strength V_{ult}.

- For the steel stud backup:

$V_r \geq V_f = 0.65$ kN
thus the ultimate strength can be determined as follows
$$V_{ult} = \frac{V_r}{\phi} = \frac{0.65}{0.6} = 1.08 \text{ kN}$$
This value is slightly higher than the minimum of 1 kN prescribed by CSA A370-04 and governs.

- For the concrete block backup:

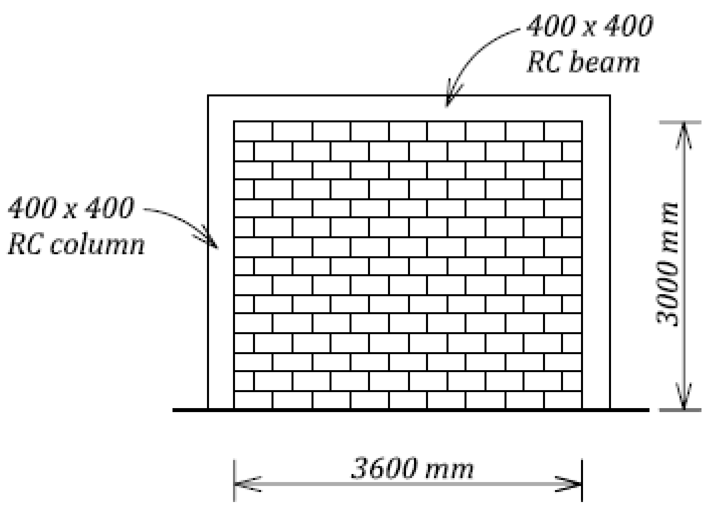
$V_r \geq V_f = 0.42$ kN
thus the ultimate strength can be determined as follows
$$V_{ult} = \frac{V_r}{\phi} = \frac{0.42}{0.6} = 0.7 \text{ kN}$$
This value is less than the minimum of 1 kN, so the minimum requirement governs.

EXAMPLE 8: Seismic design of a masonry infill wall

A single-storey reinforced concrete frame structure is shown in the figure below. The frame is infilled with an unreinforced, ungrouted concrete block wall panel that is in full contact with the frame. The wall is built using 190 mm hollow blocks and Type S mortar.

a) Model the infill as an equivalent diagonal compression strut. Determine the strut dimensions according to CSA S304.1 assuming the infill-frame interaction.

b) Assuming that the infill wall provides the total lateral resistance, determine the maximum lateral load that the infilled frame can resist. Consider the following three failure mechanisms: strut compression failure, diagonal tension resistance, and sliding shear resistance.



Given:

E_f =25000 MPa concrete frame modulus of elasticity

f'_m = 9.8 MPa hollow block masonry, from 15 MPa block strength and Type S mortar (Table 4, CSA S304.1)

SOLUTION:

a) Find the diagonal strut properties.

- Key properties for the masonry wall and the concrete frame

Concrete frame:

E_f =25000 MPa

Beam and column properties:

$$I_b = I_c = \frac{(400)^4}{12} = 2.133 * 10^9 \text{ mm}^4$$

Masonry:

$E_m = 850 f'_m = 850 * 9.8 = 8330$ MPa

Effective wall thickness (face shells only):

$t_e = 75$ mm (Table D-1, 200 mm hollow block wall)

- Diagonal strut geometry (see Section 2.6.2 and S304.1 Cl.7.13)

$h = 3000$ mm

$l = 3600$ mm

Find θ (angle of diagonal strut measured from the horizontal):

$$\tan(\theta) = \frac{h}{l} = \frac{3000}{3600} = 0.833 \qquad \theta = 39.8^\circ$$

Length of the diagonal strut:

$$l_s = \sqrt{h^2 + l^2} = \sqrt{3000^2 + 3600^2} = 4686 \text{ mm}$$

Find the strut width (see Figure 2-36):

$$\alpha_h = \frac{\pi}{2}\left(\frac{4E_f I_c h}{E_m t_e \sin 2\theta}\right)^{1/4} = \frac{\pi}{2}\left(\frac{4*25000*2.133*10^9*3000}{8330*75*\sin(2*39.8^\circ)}\right)^{1/4} = 1587$$

$$\alpha_L = \pi\left(\frac{4E_f I_b l}{E_m t_e \sin 2\theta}\right)^{1/4} = \pi\left(\frac{4*25000*2.133*10^9*3600}{8330*75*\sin(2*39.8^\circ)}\right)^{1/4} = 3322$$

Strut width:

$$w = \sqrt{{\alpha_h}^2 + {\alpha_L}^2} = \sqrt{(1587)^2 + (3322)^2} = 3682 \text{ mm}$$

Effective diagonal strut width w_e for the compressive resistance calculation should be taken as the least of (Cl.7.13.3.3)

$w_e = w/2 = 3682/2 = 1841$ mm

or

$w_e = l_s/4 = 4686/4 = 1172$ mm

thus

$w_e = 1172 \approx 1170$ mm

The design length of the diagonal strut l_d should be equal to (Cl.7.13.3.5)

$l_d = l_s - w/2 = 4686 - 3682/2 = 2845$ mm

b) Determine the maximum lateral load which the infilled frame can resist assuming that the infill wall provides the total lateral resistance.

- Diagonal strut: compression resistance (Cl.7.13.3.4 and Section 2.6.2)

The compression strength of the diagonal strut $P_{r\max}$ is equal to the compression strength of masonry times the effective cross-sectional area, that is,

$$P_{r\max} = (0.85\chi\phi_m f'_m)\cdot A_e$$

where

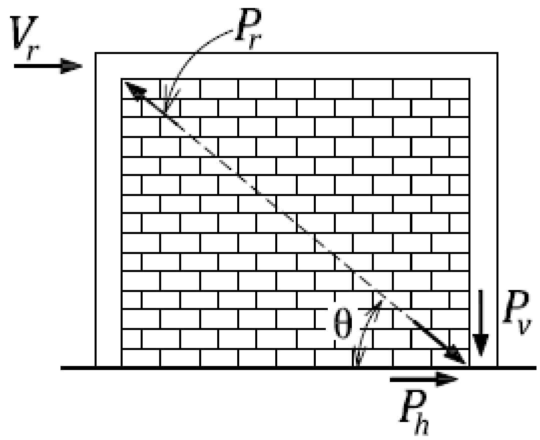
$\phi_m = 0.6$

$\chi = 0.5$ the masonry compressive strength parallel to bed joints

$A_e = t_e * w_e = 75*1170 = 87750$ mm^2 the effective cross-sectional area

$P_{r\max} = 0.85 * 0.5 * 0.6 * 9.8 * 87750 = 219.3$ kN

The corresponding lateral force is equal to the horizontal component of the strut compression force P_h, that is, (see the figure below)
$P_h = P_{r\max} * \cos(\theta) = 219.3 * \cos(39.8) = 168.0$ kN



Before proceeding with the design, slenderness effects should also be checked. First, the slenderness ratio needs to be determined as follows (Cl.7.7.5):

$$\frac{k * l_d}{t} = \frac{1.0 * 2845}{190} = 15.0$$

where
$k = 1.0$ assume pin-pin support conditions
$l_d = 2845$ mm design length for the diagonal strut
$t = 190$ mm overall wall thickness
The strut is concentrically loaded, but the minimum eccentricity needs to be taken into account, that is,
$e_1 = e_2 = 0.1 * t = 19$ mm
Since

$$\frac{k * l_d}{t} = 15.0 > 10 - 3.5 e_1/e_2 = 6.5 \text{ and } \frac{k * l_d}{t} < 30.0$$

the slenderness effects need to be considered.

The critical axial compressive force for the diagonal strut P_{cr} will be determined according to S304.1 Cl.7.7.6.3 as follows:

$$P_{cr} = \frac{\pi^2 \phi_{er} E_m I_{eff}}{(1 + 0.5\beta_d)(kl_d)^2} = 1380 \text{ kN}$$

where
$\phi_{er} = 0.65$
$\beta_d = 0$ assume 100% seismic live load
$E_m = 8330$ MPa modulus of elasticity for masonry
$I_{eff} = 0.4 I_o = 209 * 10^6$ mm^4
where

$I_o = \dfrac{1170 * \left[190^3 - (190 - 75.4)^3\right]}{12} = 522 * 10^6$ mm^4 moment of inertia of the effective cross-sectional area based on the effective diagonal strut width $w_e = 1170$ mm and the effective wall thickness $t_e = 75.4$ mm (face shells only).

Since
$P_{r\max} = 219.3$ kN < $P_{cr} = 1380$ kN
it follows that compression failure governs over buckling failure.

- The diagonal tension shear resistance (see Section 2.3.2 and CSA S304.1 Cl.10.10.1).

Find the masonry shear resistance (V_m):

$b_w = 190$ mm overall wall thickness

$d_v \approx 0.8 l_w = 2880$ mm effective wall depth

$\gamma_g = 0.5$ ungrouted wall

$P_d = 0$ (ignore self-weight)

$v_m = 0.16\sqrt{f'_m}$ = 0.5 MPa

$V_m = \phi_m (v_m b_w d_v + 0.25 P_d)\gamma_g$ = 0.6(0.5*190*2880+0)*0.5 ≈ 82.0 kN

This is a squat shear wall because $\dfrac{h_w}{l_w} = \dfrac{3000}{3600} = 0.83 < 1.0$. In this case, there is no need to find the maximum permitted shear resistance per S304.1 Cl.10.10.1.3 $\max V_r$ because it is not going to control for an unreinforced wall without gravity load.

- Sliding shear resistance (see Section 2.6.1 and Cl.7.10.4)

$V_{rs} = 0.16\phi_m \sqrt{f'_m} A_{uc} + \phi_m \mu P_1$

The factored in-plane sliding shear resistance V_r is determined as follows.

μ = 1.0 for a masonry-to-masonry or masonry-to-roughened concrete sliding plane

$A_{uc} = t_e \cdot d_v = 75 * 2880 = 216000$ mm^2 uncracked portion of the effective wall cross-sectional area

The compressive force in masonry acting normal to the sliding plane is normally taken as P_d plus an additional component, equal to 90% of the factored vertical component of the compressive force resulting from the diagonal strut action P_v (see the figure on the previous page).

$P_1 = P_d + 0.9 * P_v$

where

$P_v = V_{rs} * \tan(\theta)$

thus

$P_1 = 0 + 0.9 * V_{rs} \tan(\theta)$

The sliding shear resistance can be determined from the following equation

$V_{rs} = 0.16\phi_m \sqrt{f'_m} A_{uc} + \phi_m \mu (0.9 * V_{rs} \tan(\theta))$

or

$$V_{rs} = \frac{0.16\phi_m \sqrt{f'_m} A_{uc}}{1-\phi_m * \mu * 0.9 * \tan(\theta)} = \frac{0.16 * 0.6 * \sqrt{9.8} * 216000}{1-0.6*1.0*0.9*\tan(39.8^\circ)} = 118.0 \text{ kN}$$

- Discussion

It is important to consider all possible behaviour modes and identify the one that governs in this design. The following three lateral forces should be considered:

a) $P_h = 168$ kN shear force corresponding to the strut compression failure

b) $V_m = 82$ kN diagonal tension shear resistance

c) $V_{rs} = 118$ kN sliding shear resistance

It could be concluded that the diagonal tension shear resistance governs, however once diagonal tension cracking takes place, the strut mechanism forms. Therefore, the maximum shear force developed in an infill wall corresponds either to the strut compression resistance or the sliding shear resistance (see the discussion in Section 2.6.2). In this case, sliding shear resistance governs and so $V_{r\max} = V_{rs} = 118$ kN.

It should be noted that the maximum shear force developed in the infill $V_{r\max}$ will be transferred to the adjacent reinforced concrete columns, which need to be designed for shear. This is not the scope of the masonry design, however the designer should always consider the entire lateral load path and the force transfer between the structural components.