Trading Volatility As An Asset Class

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Summary

Volatility is a useful trading hedge against all kinds of disasters. How can you trade it?

Calls and puts don't quite do it. Though calls and puts are sensitive to volatility, they are not sensitive *only* to volatility.

How can you do better?

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WHY TRADE VOLATILITY?

Volatility is the simplest measure of a stock's riskiness or uncertainty.
Different types: realized volatility, implied volatility, local volatility...

Realized Volatility

The realized daily volatility σ_d of an equity index S_i over a period of N days is the square root of the variance of the daily returns r_i:

$$r_i \approx \frac{\Delta S_i}{S_i} \qquad \sigma_d^{\;2} = \frac{1}{N}\sum_i (r_i)^2 - \left(\frac{1}{N}\sum_i r_i\right)^2$$

Stock returns are roughly random and normal; variance grows ~ return time.

$$\Delta S \approx \sigma S \sqrt{\Delta t}$$

$$\sigma_{annual} \sim 16 \times \sigma_{daily}$$

Similarly for currencies, commodities, interest rates, volatility itself,

Implied Volatility

Black-Scholes: the fair price of a stock option depends on its future realized volatility.

$$C_{BS} = C(S, K, t, T, r, \sigma)$$

Black-Scholes is both a model and a quoting mechanism for options prices.

Implied volatility Σ is the value of the volatility σ you have to insert into the Black-Scholes formula to make it match the market price of an option.

You can think of it as the market's expectation of future realized volatility, plus a spread.

I

WHY TRADE VOLATILITY?

Bonds and Options /Yields and Volatilities

Bonds	Options
Interest rates are the parameters people use to quote bond prices.	Volatilities are the parameters people use to quote options prices
Realized daily interest rates: actual short-term interest rates	Realized daily volatility: the actual volatility of an index
Yield to maturity of a bond: the average of the future realized rates that make that bond price fair. It's the implied yield based on price.	Implied volatility of an option: the average of future realized volatilities that make the options price fair, based on Black-Scholes.
Forward rates: the future realized rates, moment by moment, that must come to pass to make current yields of all liquid bonds fair.	Local (forward) volatilities: the future realized index volatilities, index level by index level and moment by moment, that must come to pass to make current implied volatilities fair.

Why Trade Volatility?

Attractive characteristics:

- It grows when uncertainty increases.
- It reverts to the mean.
- It goes up and tends to stay up when most assets go down.

Types of volatility trading:

- Speculative trading of volatility levels in various markets.

  Index vs. stock, foreign vs. domestic, short-term vs. long-term, currencies, rates,...

  You need views about future uncertainty to trade it.
- Trading the spread between realized and implied volatility levels.
- Hedging implicit volatility exposure.

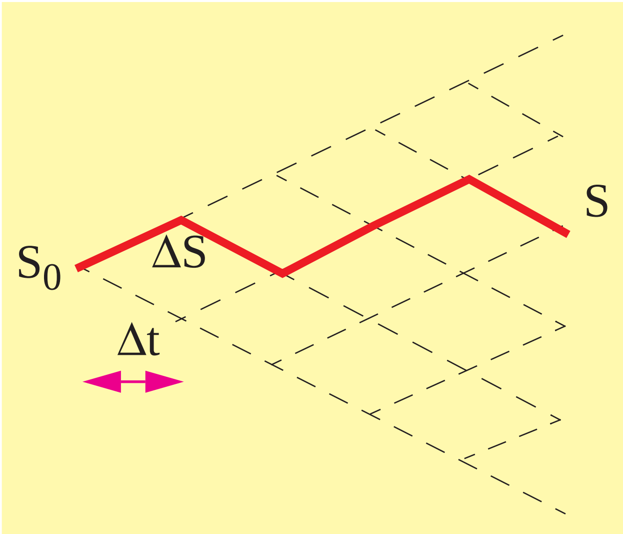
  Hedge funds and risk arbitrageurs are often implicitly short volatility. They often take short positions in the spread between stocks of companies planning to merge, assuming that the spread will narrow if the merger takes place. If overall market volatility increases, these mergers are less likely to occur, and the spread may widen.

II

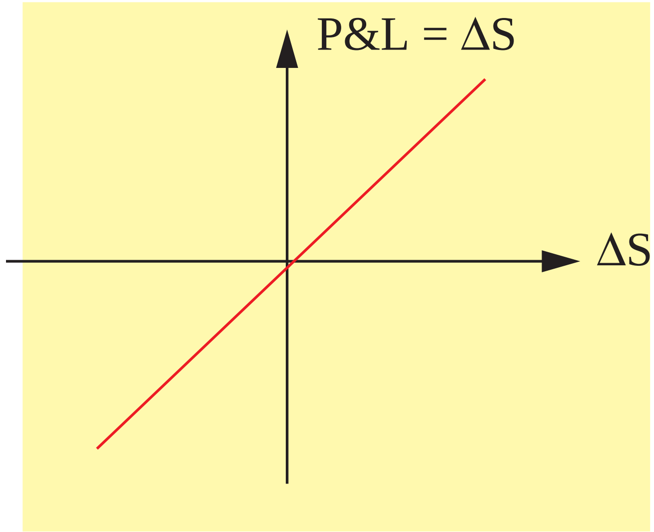
VOLATILITY TRADING WITH OPTIONS

A Linear Security: A stock or index

ASSUMED STOCK EVOLUTION:



P&L OF A STOCK POSITION:



- If you own a stock or index, you make money if it goes up, lose if it goes down.
- You have a linear position in ΔS over the next instant Δt.
- As an investor, how do you profit no matter whether the index goes up or down?

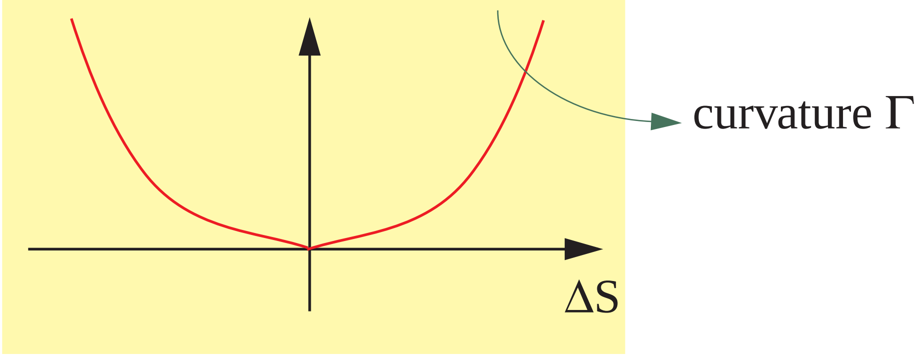
II

VOLATILITY TRADING WITH OPTIONS

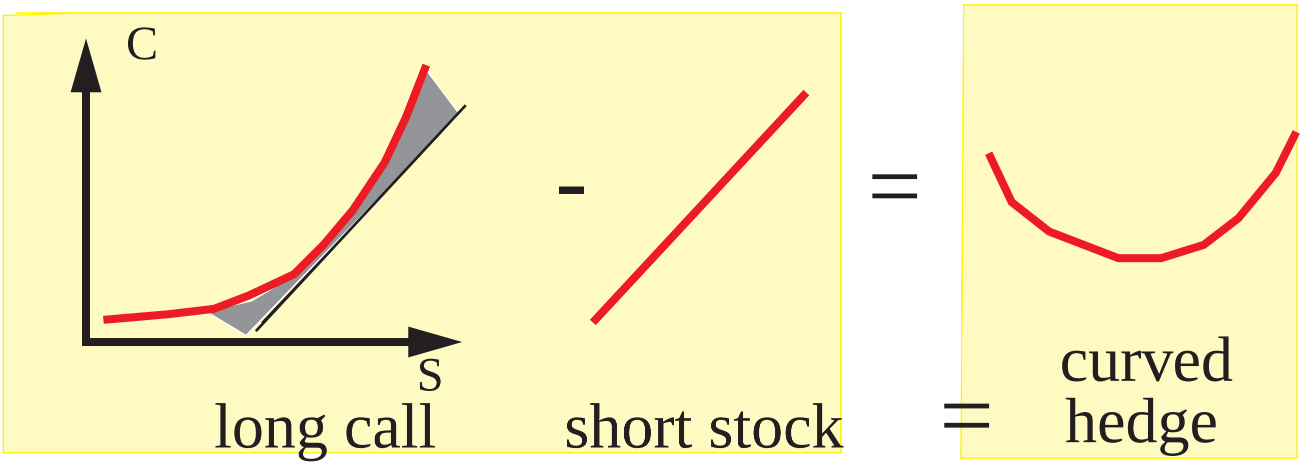
A Curved Security

To make money irrespective of direction, you want a security which gives you a curved P&L: a quadratic position in $(\Delta S)^2$:

$$P\&L = (1/2)\Gamma(\Delta S)^2$$

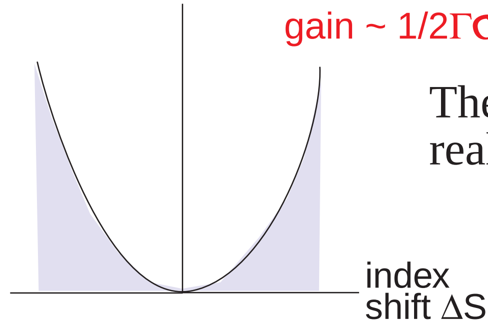


To get curvature: delta-hedge away the linear part of a call option.



P&L of Delta-Hedged Options

gain ~ $1/2\Gamma\sigma^2 S^2 \Delta t$

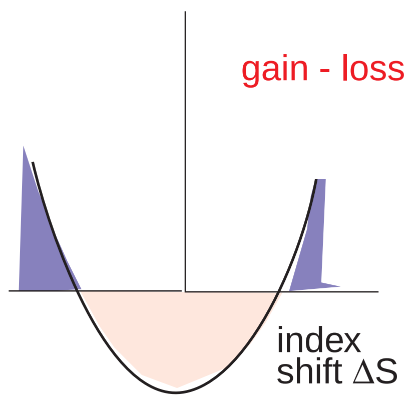


The gain due to a move ΔS with realized vol σ

loss ~ $(1/2)\Sigma^2 S^2 \Delta t$

The loss in an instant Δt if expected vol is Σ

gain - loss ~$(1/2)\Gamma[\sigma^2 - \Sigma^2]S^2 \Delta t$



A hedged position makes money if realized vol exceeds implied vol, with magnitude depending on option's curvature Γ and stock price S^2

P&L Depends on Realized Vol vs. Implied Vol

$$\text{Net P\&L} = \frac{1}{2}\int \Gamma S^2\left(\sigma^2 - \Sigma^2\right)\Delta t$$

Long options: you make money if $\sigma > \Sigma$

Short options: you make money if $\sigma < \Sigma$

III

Options Trading: What Can Go Wrong

III

Options Trading: What Can Go Wrong

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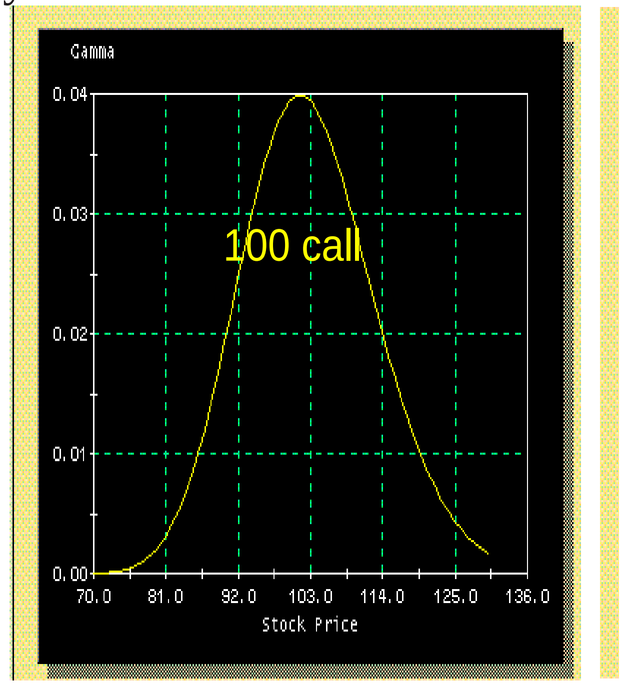
OPTIONS TRADING: WHAT CAN GO WRONG

1. It's Not A Clean Bet on Volatility Alone

$$\text{Net P\&L} = \frac{1}{2}\int \Gamma S^2\left(\sigma^2 - \Sigma^2\right)\Delta t$$

ΓS^2 is irritating. The Γ (Gamma) (Curvature) of an option as stock price S varies sharply:



If the stock price moves away, you get very little bang for your option buck.

2. Delta-Hedging In Practice Is Risky

Delta-hedging assumes smooth stock movements, continuous hedging, no transactions costs, a known future volatility.

Real markets violate Black-Scholes assumptions because of:

- Jumps.
- Transaction costs.
- Stochastic volatility: the future realized volatility which determines your hedge ratio is not known.
- You cannot really hedge continuously; you must hedge discretely.

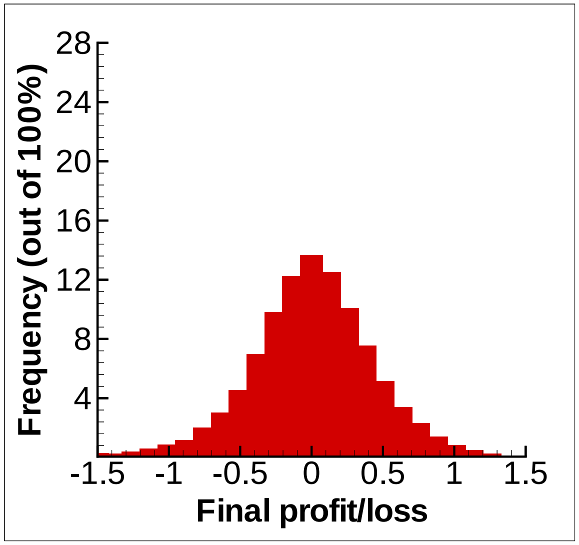
All of these imperfections may overwhelm any theoretical gain.

Let's look at just one example -- discrete hedging.

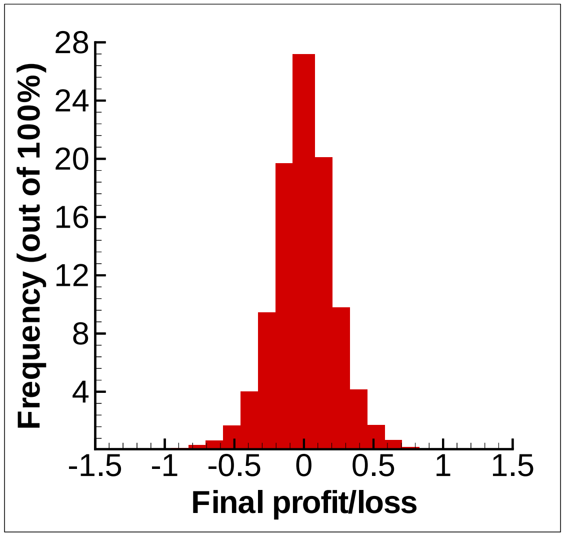
Example: Error When Hedging is Discrete

Perfect Black-Scholes world; hedge N times for 1-month ATM put with 20% realized vol. The Black-Scholes value is 2.512 dollars.

Monte Carlo simulation of the P&L:



N=21 (roughly daily)
Mean=0
Standard deviation=0.41
About 16% error, 4 vol pts



N=84
Mean=0
Standard deviation=0.20
About 8% error, 2 vol pts

Your P&L isn't guaranteed unless you hedge continuously. But that introduces large transactions costs and shifts the expected return.

It's not easy to make money this way.

IV

THE VOLATILITY SMILE VIOLATES BLACK-SCHOLES

The “smile” is the characteristic variation of implied volatility with strike and expiration. It is inconsistent with the Black-Scholes model everyone uses.

Implied Volatility Σ Violates Black-Scholes

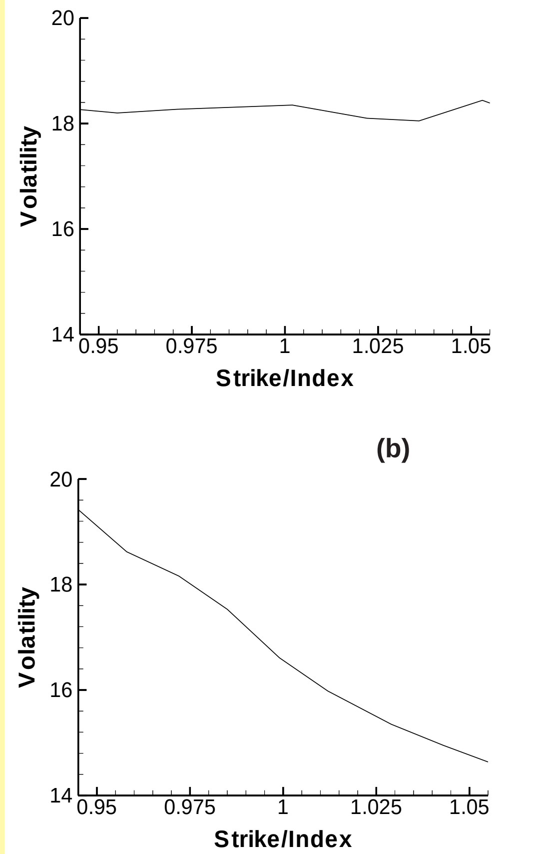
- Implied volatility is the single value of the volatility you have to insert into the Black-Scholes model to match the market price of an option.
- It is the future volatility the index must have to make the Black-Scholes price fair.
- If the Black-Scholes model is correct, all options would have the same implied volatility.
- That isn’t the case.

Black-Scholes is wrong in principle, not just in practice, and therefore hedging is even more difficult.

Pre- and Post-Crash Implied Volatilities:

Since the ‘87 crash there has been a persistent skewed structure in Black-Scholes implied volatilities in most world equity option markets.

Representative implied volatility skews of S&P 500 options. (a) Pre-crash. (b) Post-crash. Data taken from M. Rubinstein, “Implied Binomial Trees” *J. of Finance*, 69 (1994) pp 771-818.



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THE VOLATILITY SMILE VIOLATES BLACK-SCHOLES

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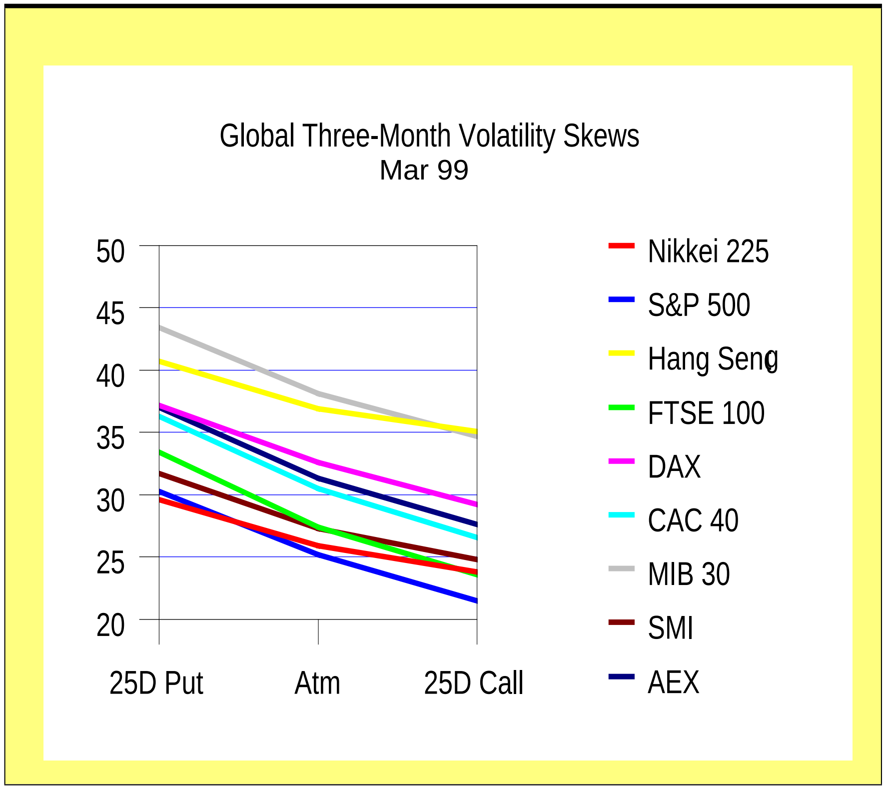
THE VOLATILITY SMILE VIOLATES BLACK-SCHOLES

A Persistent Negative Global Skew/Smile

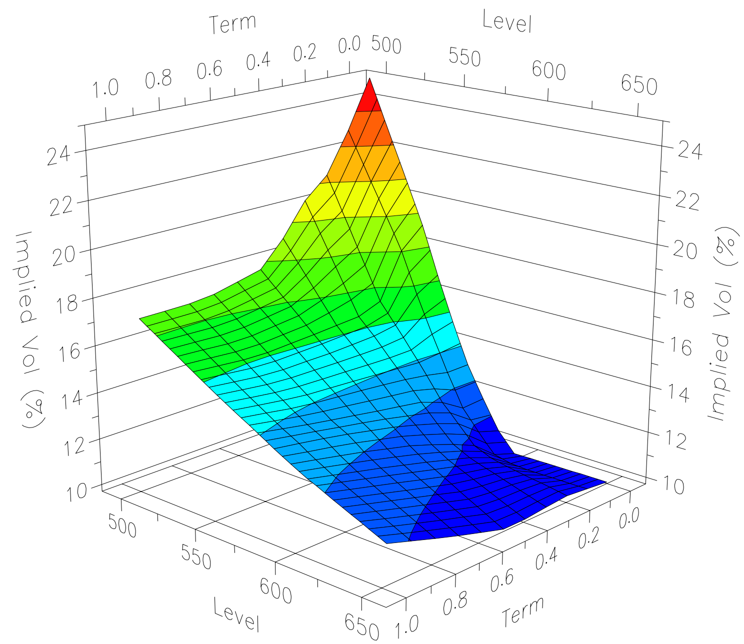
A persistent large skew, almost linear, and inconsistent with Black-Scholes.



$$\Sigma(K) = \Sigma_{atm} - b(K - S_0)$$

The Implied Volatility Surface of Indexes

The implied volatility surface for S&P 500 index options as a function of strike level and term to expiration on September 27, 1995.

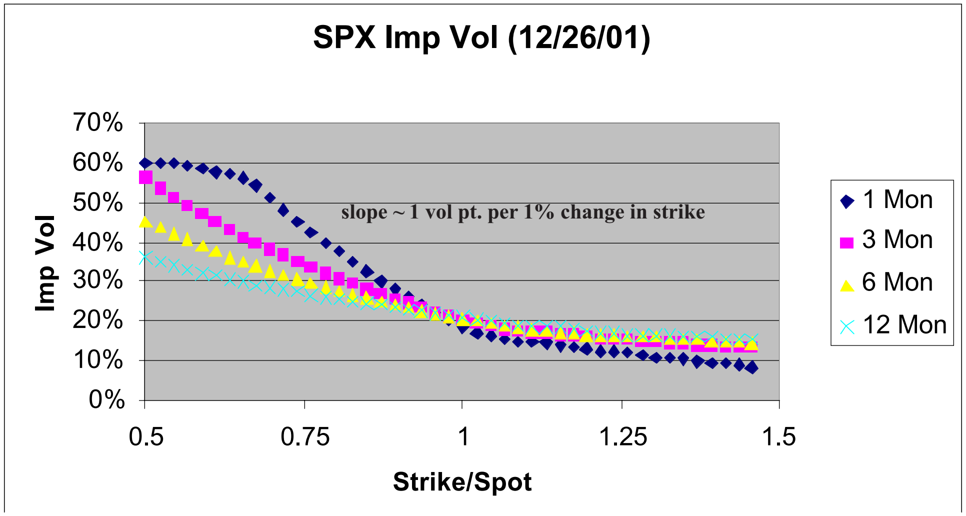


- Out-of-the-money puts always have higher implied volatilities than out-of-the-money calls.
- Short-term volatilities are usually more volatile.
- Implied volatility is usually greater than recent historical volatility.

IV

THE VOLATILITY SMILE VIOLATES BLACK-SCHOLES

More recent S&P 500 smiles



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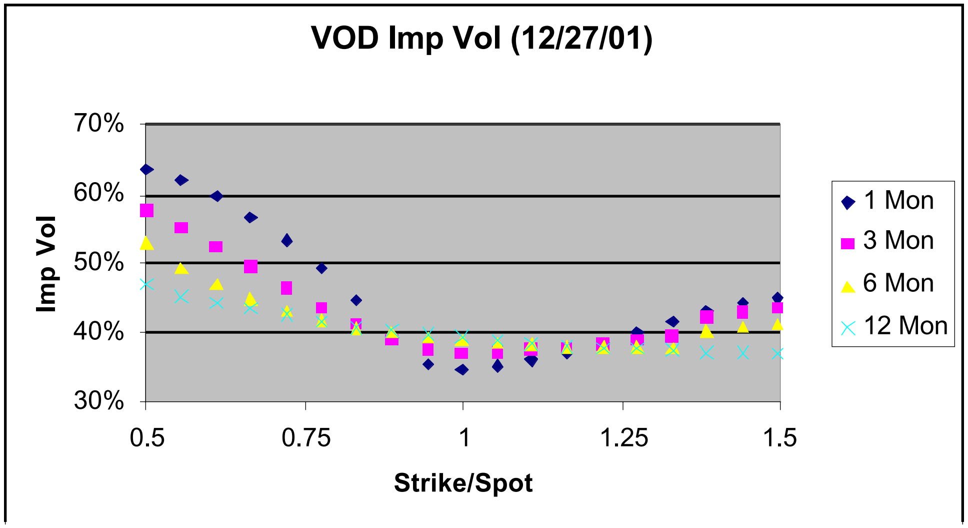
THE VOLATILITY SMILE VIOLATES BLACK-SCHOLES

Single stock smile



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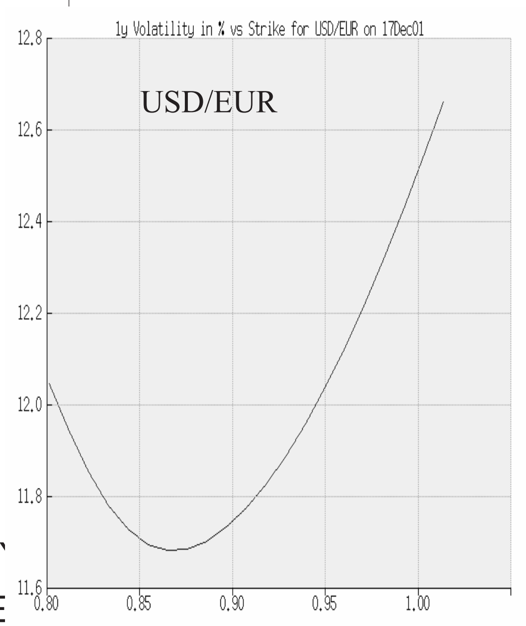
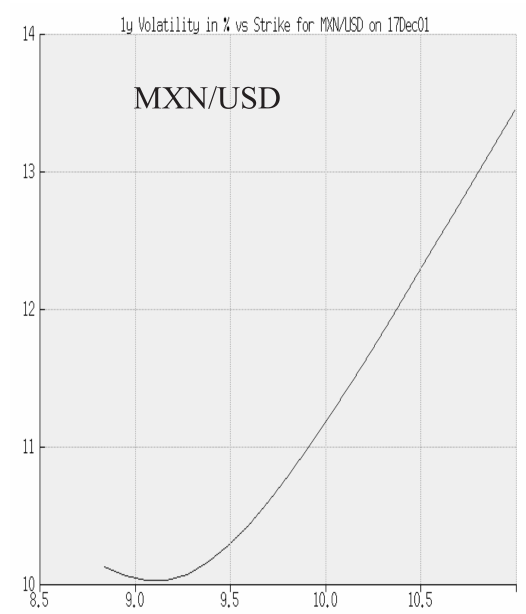
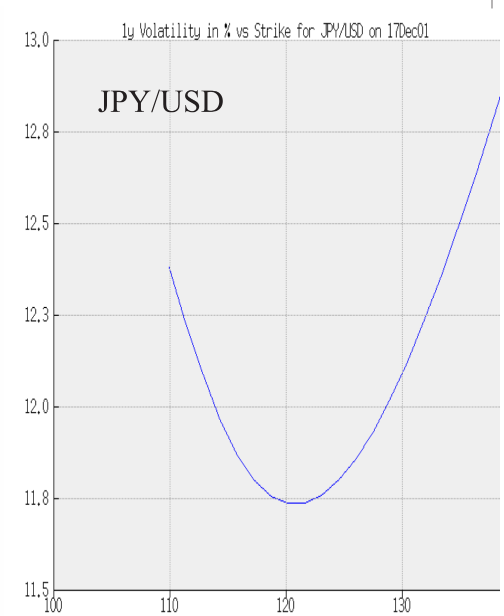
Some currency smiles....







ATM Strike = 0.90 9.85 123.67

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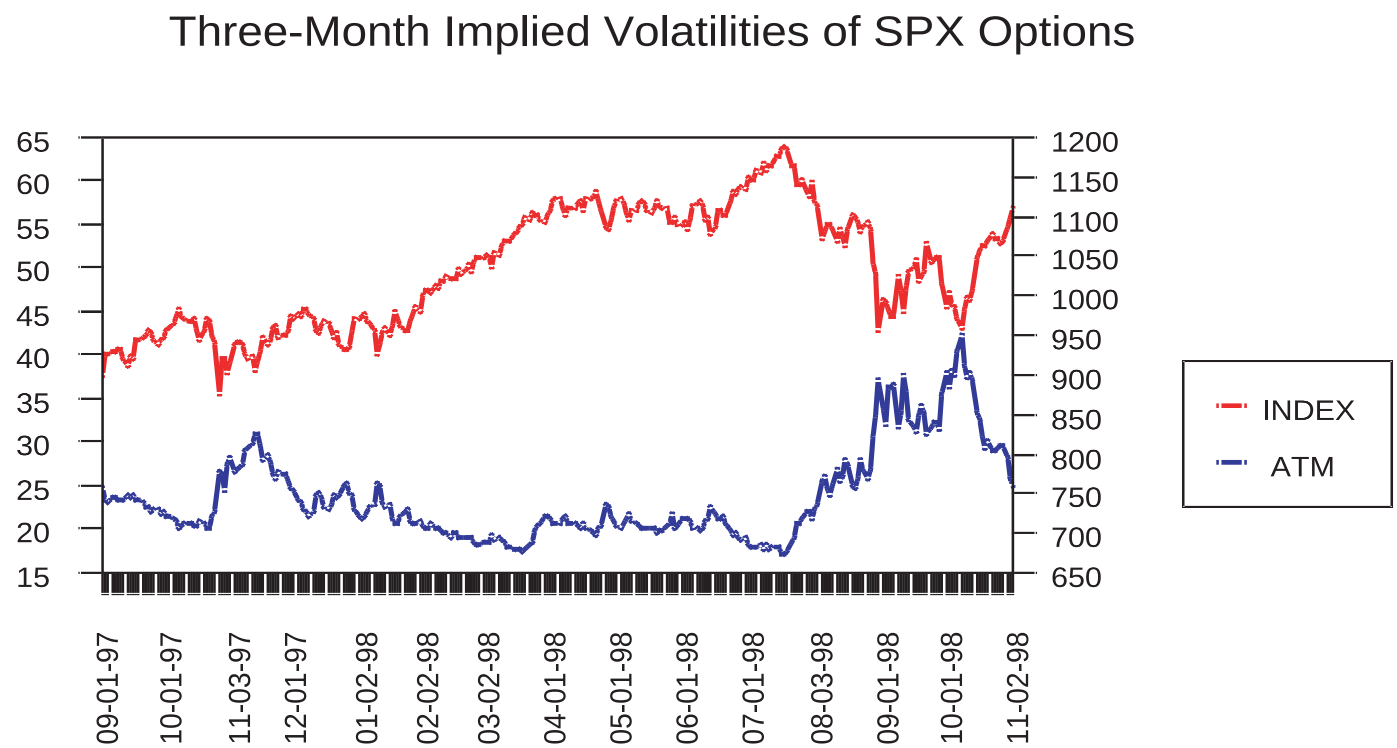
Negative Correlation Between Implied Vols and Index Levels



But be careful - ATM vol isn't something you own. You own a specific strike vol.

IV

THE VOLATILITY SMILE VIOLATES BLACK-SCHOLES

Patterns of Change in the Volatility Surface

Volatility surfaces fluctuate in modes:

$$\Delta\Sigma = \beta_1 \text{ (volatility level mode) } + \beta_2 \text{ (term structure mode) } + \beta_3 \text{ (skew mode) } \quad \vdots$$
$$+ \, other \;\; modes$$

Modes, or factors, are movements of the entire surface.

As with interest rates, there are parallel shifts, “steepening”, and “twists.”

Modes are useful when:

- They can be understood intuitively.
- Few modes explain most of the variation.
- Modes are historically stable.

One finds about 85% of moves are parallel, 10% changes in slope with respect to time, and 5% related to out-of-the-money short term puts.

V

WHAT CAUSES THE SMILE?

V

WHAT CAUSES THE SMILE?

Causes of The Smile

Behavioral causes:

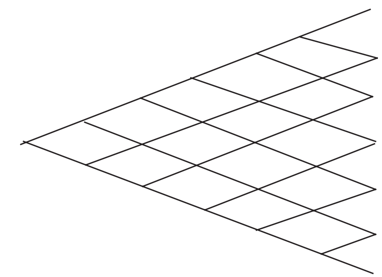
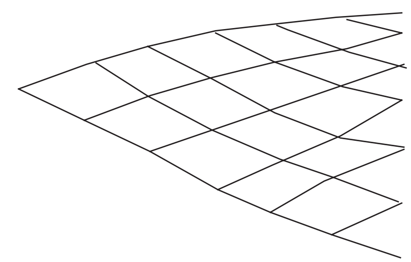
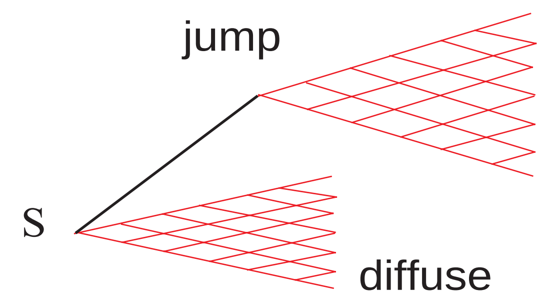
- Supply and demand - purchases of collars
- Expectation of changes in volatility
- Fear of crashes (down for equities, up for gold
  - After a crash, realized and implied volatilities will be higher
  - correlation of individual stocks in a jump down increases
- Level- and time-dependent effects
  - resistance levels in stocks
  - support levels in currencies
  - change of volatility behavior at low interest rates

Also:

- Fat tails in distributions
- Leverage effects
- Transactions costs?
- Uncertain future volatility

All of these effects make Black-Scholes wrong: the underlier doesn't carry out simple Brownian motion.

Models for the Smile

Classic Black-Scholes: constant volatility	
Local volatility models: correlated volatility	
Stochastic volatility models:	random volatility
Jump-diffusion models: small probability of a large jump	

Which model you use depends on what market you deal with.

- Currencies tend to have stochastic volatilities.
- Interest rates have volatilities that depend on interest levels.
- Stock markets tend to jump in the short run, diffuse over longer times.
-

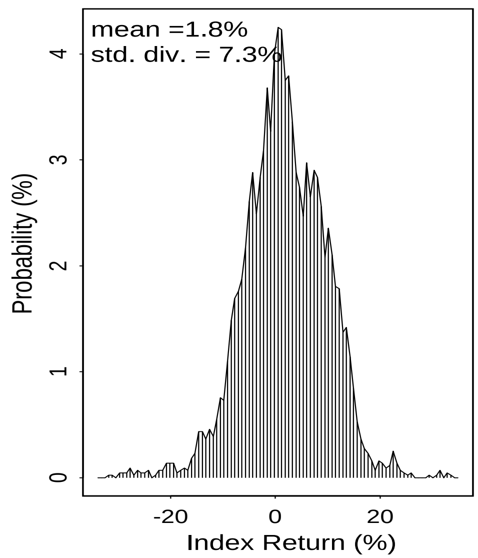
There is no single correct replacement for Black-Scholes. It's hard to test options models under the best of circumstances.

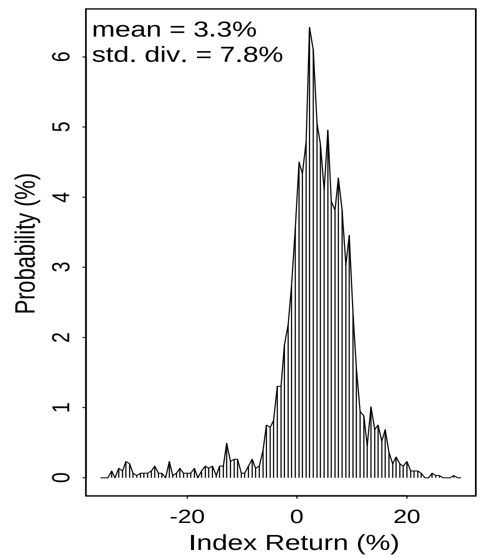
VI

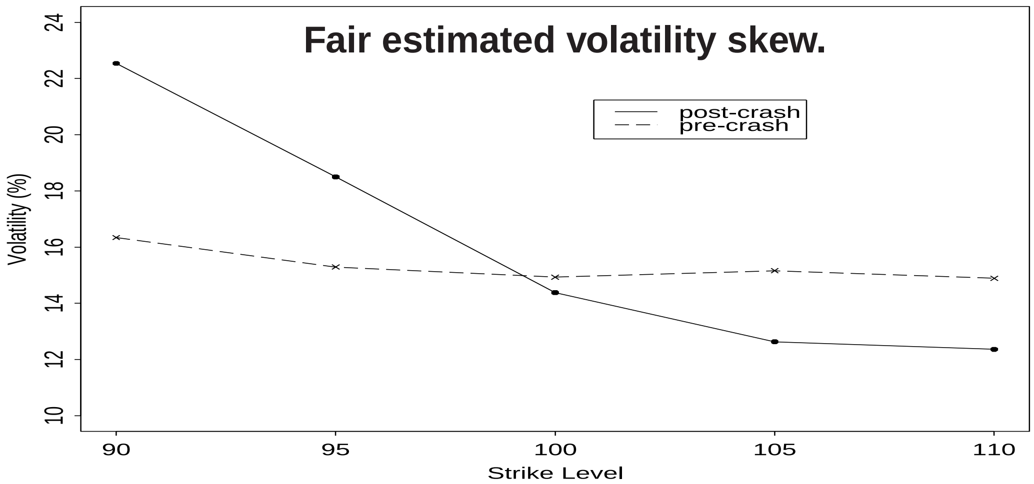
Is The Smile Fair?

Pre- and Post-Crash SPX Return Distributions

Three-month, S&P 500 index, observed return distributions.
(a) pre-1987 crash (1/70 to 1/87); (b) post-1987 crash (6/87 to 6/99)







VI

IS THE SMILE FAIR?

VII

TRADING & PRICING VOLATILITY USING VOLATILITY SWAPS

The cleanest and easiest way to trade volatility is through a volatility swap, which is a forward contract on realized volatility.

Swaps have become very popular because dealers have access to a theory for pricing them relative to the options market.

The theory provides a basis for defining volatility indexes like the VIX in terms of the price of a basket of options that can be traded, and for creating your own volatility swaps.

Analogy: Credit Default Swap Market

Corporate Bonds	Hedged Options
Bond prices are influenced by interest rates and credit spreads	Options prices are influenced by stock prices and volatility
Credit default swaps allow you to simply bet on credit spreads	Volatility swaps let you trade pure volatility.

Why isn't there a market for volatility alone?

Volatility Contracts: Volatility and Variance Swaps

A Volatility Swap is a forward contract on realized volatility:

Payoff: $(\sigma_R - K_{vol}) \times N$ where N is the notional amount.

σ_R: realized volatility realized by the index until expiration;

K_{vol}: previously agreed upon "delivery" volatility.

A **variance swap** is a forward contract on realized variance. It pays

$$\left(\sigma_R^2 - K_{var}\right) \times N$$

Must specify the precise method for calculating realized volatility, the source and observation frequency of prices, and the annualization factor.

Variance swaps are easier to price and hedge using options.

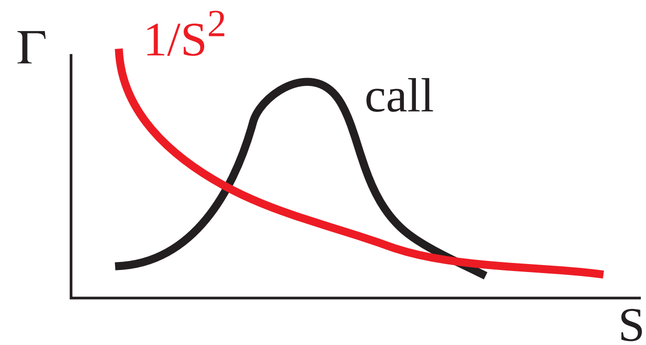
A dealer will sell you a variance swap. You can trade volatility in one market or sector against another easily, without the hassle and errors of hedging options. But you want to understand how to price it!

Dealers create a variance swap by hedging a Log Contract

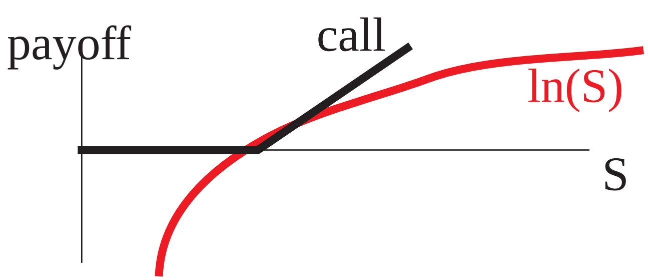
Trading a single option: Net P&L $= \frac{1}{2}\int \Gamma S^2 \left(\sigma^2 - \Sigma^2\right) \Delta t$

An option whose $\Gamma S^2 = 1$ would exactly earn the realized volatility over the life of the contract, with a delivery price equal to Σ^2.

- What kind of option has a $\Gamma \sim 1/S^2$?



- An option whose payoff is the natural logarithm of S: – ln(S)



VII

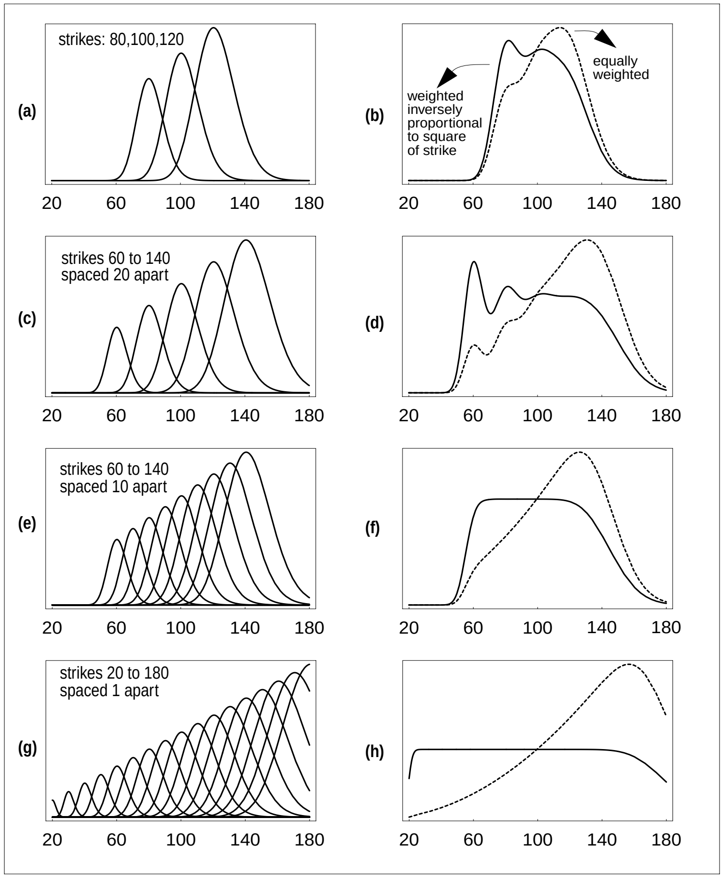
TRADING & PRICING VOLATILITY USING VOLATILITY SWAPS

Dealers Can Create a Log Contract Out of A Basket of Options with Many Different Strikes

A portfolio of vanilla options with weights inversely proportional to their strike squared can replicate the payoff of a Log Contract. It will be equally sensitive to volatility at all spot level S.

A $1/K^2$ density of puts and calls will give the same payoff as a Log Contract.

It will be equally sensitive to volatility at all spot levels



VII

TRADING & PRICING VOLATILITY USING VOLATILITY SWAPS

Pricing of Variance Swaps

- Dealers replicate the log contract by buying and hedging a portfolio of puts and calls (and a forward contract).
- The fair price of the variance swap is given by the market price of the basket of puts and calls, a price which depends on the smile.

You can create your own variance swaps like this to do vol arbitrage.

Difficulties to be aware of

- You need a continuum of puts and calls of all strikes to replicate it exactly.
- In practice you cannot buy very out-of-the-money strikes. So, if the stock price moves too far, your replication will fail.
- If the stock jumps rather than moves smoothly, there are additional replication failures

Volatility swaps are more complex.

Volatility is the square root of variance, and is a more complex derivative of variance. Its value depends not just on volatility, but on the volatility of volatility, and you have to dynamically hedge it by trading variance swaps as the underlier. This is possible too, but needs a model for the volatility of volatility